



Maths GCSE methods

PART 1 – NUMBERS

At drawmaths, **we learn maths through drawings**, and other visual representations. This resource presents visual methods to understand and to use numbers. Drawing and visualising give meaning to maths concepts. It allows to link abstract concepts to concrete examples. It makes learning relevant and allows us to get a deeper understanding.

This resource is aimed at students preparing for **GCSE examinations**, both foundation and higher level.

This resource is evolving, and we would be **keen to receive feedback and ideas** from teachers, parents, and students alike, so don't hesitate to get in touch.

Our next steps include adding:

- a section on **truncation**
- **real-life examples**
- **problems and exam questions** with **model solutions**
- **demo videos** and QR codes linking to the videos
- this content to the **drawmaths website** and videos onto **drawmaths YouTube channel**

In the longer term, we are hoping to transfer the content to **Moodle using HSP interactive presentations and activities**. We are planning to create **booklets for the rest of the GCSE Maths curriculum**: algebra, graphs, ratio, geometry, angles, trigonometry probability, stats etc.

Future links of interest (these will be made live asap):

- Demo videos: www.youtube.com/@DRAWmaths
- Website: <https://www.drawmaths.com/>
- Email: info@drawmaths.com

Legend

Methods are in boxes



Definitions are next to an owl



Higher content has an H & arrow

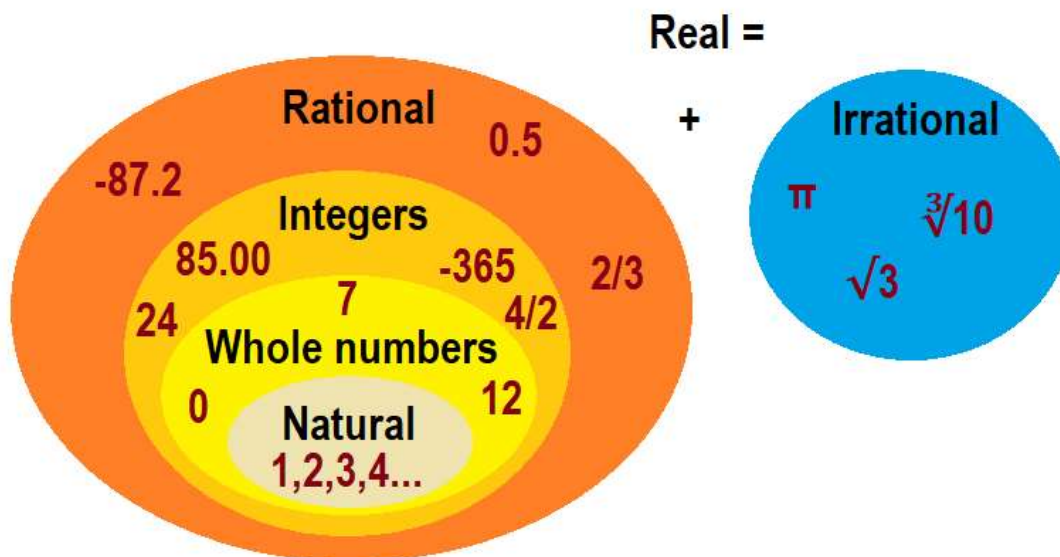


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1 Numbers and place value

1.1 Types of Number



- **Natural (N)** = numbers used for counting, all positive integers, e.g.
 - 1
 - 2
 - 3
 - 4...
- **Whole numbers** are the same as natural numbers, but they include zero. They do not include negative numbers.
 - 0
 - 7
 - 12
- **Integer (Z)** = positive and negative whole numbers and zero, e.g.
 - - 365
 - 0
 - 24
 - $\frac{4}{2} = 2$
 - $85.00 = 85$
- **Real (R)** = all the numbers on the number line, e.g. 12, $\sqrt{21}$, 0, -0.2. Real numbers can be:
 - **Rational (Q)** = numbers that can be expressed as a fraction $\frac{a}{b}$ where a and b are two integers (with $b \neq 0$). They include terminating decimals and recurring decimals, e.g.
 - $\frac{2}{3}$
 - $-87.2 = \frac{-872}{10} = \frac{-43}{5}$
 - $0.5 = \frac{1}{2}$
 - $0.3\dot{1} = \frac{14}{45}$
 - **Irrational (I)** = numbers that cannot be expressed as a fraction $\frac{a}{b}$ where a and b are two integers. They include non-terminating, non-recurring decimals, e.g.
 - $\sqrt{3}$
 - $\sqrt[3]{10}$
 - π

1.2 Our number system and place value

1.2.1 A number system in base 10



Our number system is in **base 10** and we assign a **place value** to the digits in a number.

- each digit can be 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 i.e. 10 possibilities.
- each position (or column) has a value in **powers of ten** – we call this a **place value**.

The **place value frame** helps visualise the value of each position:

- The **Ones column (O)** has a value of $1 = 10^0$
- The **Tens column (T)** has a value of $10 = 10^1$
- The **Hundreds column (H)** has a value of $100 = 10^2$
- The **Thousands column (Th)** has a value of $1,000 = 10^3$

Th	H	T	O
THOUSANDS	HUNDREDS	TENS	ONES
1,000	100	10	1

1.2.2 Place value

The **value of a digit** depends on its 'place' in the number, it equals **the digit times its place value**

- 1 in the Thousands column (Th) has a value of $1 \times 1,000 = \mathbf{1,000}$
- 2 in the Hundreds column (H) has a value of $2 \times 100 = \mathbf{200}$
- 4 in the Tens column (T) has a value of $4 \times 10 = \mathbf{40}$
- 7 in the Ones column (O) has a value of $7 \times 1 = \mathbf{7}$

Th	H	T	O
THOUSANDS	HUNDREDS	TENS	ONES
1,000	100	10	1
1	2	4	7

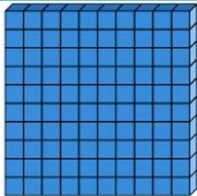

1.2.3 Place holder

As seen above, the value of a digit equals the digit multiplied by its place value. So if a digit is 0, its value is 0, whatever its place value. Still, this 0 is useful to keep the other digits in their place. It is a **place holder**.

Example: in 103:

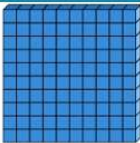
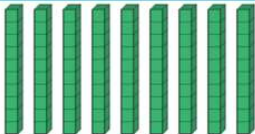
- 1 in the Hundreds column (H) has a value of $1 \times 100 = \mathbf{100}$
- 0 in the Tens column (T) has a value of $0 \times 10 = \mathbf{0}$
- 3 in the Ones column (O) has a value of $3 \times 1 = \mathbf{3}$

The 0 in the Tens has a value of 0, but it cannot be removed, otherwise the number would be 13!
The 0 in the Tens holds the 1 to its left in the Hundreds, and therefore keeping a value of 100.

H	T	O
HUNDREDS	TENS	ONES
1	0	3
		

1.2.4 It's all about 10

Each column has a place value **10 X bigger** than the column on its right. Whatever the digit in that column between 1 and 9, its value is more than the value of any digit in the column on its right (10 is bigger than 9). E.g. the 1 in the Hundreds has a value of **100**, which is more than the value of the 9 in the Tens, which is **90**.

Th	H	T	O
THOUSANDS	HUNDREDS	TENS	ONES
1,000	100	10	1
	1	0	0
			
		9	0
			

Move one place to the left and the place value is multiplied by 10.

Move to the right and it is divided by 10.

Th	,	H	T	O
THOUSANDS		HUNDREDS	TENS	ONES
1,000		100	10	1

Diagram illustrating the relationship between place values:

- Red arrows pointing left: $\times 10$ (from H to Th, from T to H, from O to T)
- Blue arrows pointing right: $\div 10$ (from Th to H, from H to T, from T to O)

We can **multiply or divide numbers by 10, 100, 1,000** using the place value frame.

Example: $23 \times 10 = 230$

Th	,	H	T	O
			2	3
		2	3	0

Diagram illustrating the multiplication of 23 by 10:

- Red arrows pointing left: $\times 10$ (from T to H, from O to T)

1.2.5 Representing decimals

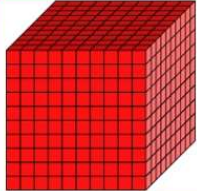
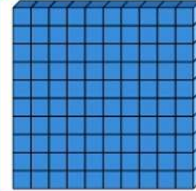
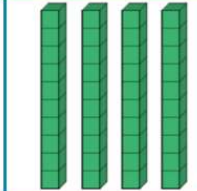
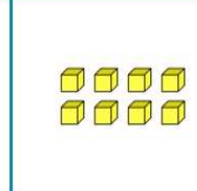
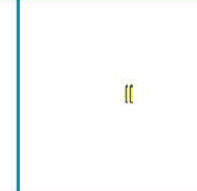
You can **use the place value frame to represent decimals**, as well as integers.

All the above holds true for digits in the fractional part and in the whole number part.

To help visualise 1 tenth, 1 hundredth of it, we use a big blue square to represent 1 (= 1 bar of chocolate)

Example: 11.482 is made of

- 1 ten (T) = $1 \times 10 = 10$
- 1 one (O) = $1 \times 1 = 1$
- 4 tenths (th) = $4 \times 0.1 = 0.4$
- 8 hundredths (h) = $8 \times 0.01 = 0.08$
- 2 thousandths (th) = $2 \times 0.001 = 0.002$

T	O	t	h	th
TENS	ONES	tenths	hundredths	thousandths
10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1,000} = 0.001$
1	1	4	8	2
				

1.2.6 Ordering numbers

ORDERING NUMBERS USING A PLACE VALUE FRAME

1. Place all numbers in a place value frame and look at the column most on the left as it holds the most value. The number with the highest digit in that column is the biggest number.
2. If some digits are equal, look at the next column down
3. Repeat until you have ordered all numbers

Example: order 11.482, 10.4, 10.6 and 10.483

1. In the Tens column, all digits are 1.
 2. In the Ones column, 11.482 has the biggest digit -1, it is the biggest number.
 3. In the tenth column, 10.6 has the biggest digit - 6, it is the next biggest.
- In the hundredth column, 10.483 has a digit of 8, while 10.4 has no digit i.e. =0, so 10.483 is the next biggest.

So, we have **11.483 > 10.6 > 10.483 > 10.4**

T	O	t	h	th
1	1	4	8	2
1	0	4		
1	0	6		
1	0	4	8	3

1.3 Rounding and estimating

Rounding a number gives an approximate value of the number, but using less 'non-zero' digits. We can round to a particular place value, to a number of decimal places or to a number of significant figures.

1.3.1 Rounding to a specific place value

ROUNDING NUMBERS USING A PLACE VALUE FRAME

- Place your number in a place value frame, point an arrow at the place value you are rounding to. Rounding works exactly the same in the whole number part and in the fractional part.
- All the digits before the arrow are **identical** (columns to the left)
- All the digits after the arrow **equal zero** (columns to the right)
- The digit that the arrow is pointing to is
 - the same** if the digit to its right is less than 5 (0, 1, 2, 3, 4)
 - one more** if the digit to its right is 5 or more (5, 6, 7, 8, 9). If originally a 9, it becomes a 10: write 0 in this place value and carry over 1 to next place value up (to the left).

T	O	.	t	h	th
TENS	ONES		tenths	hundredths	thousandths
10	1		$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1,000} = 0.001$
1	5	.	4	8	5
↓ same	↓ same		↓ 1+	↓ zero	↓ zero
1	5	.	5	0	0

↑

Example: round 15.485 to the nearest tenth

- Point an arrow to the tenth column
- All digits before the tenths column remain the same → rewrite the 1 in the Tens and the 5 in the Ones
- All digits after the tenths column equal zero → write a zero in the hundredths and in the thousandths
- Look at the tenths column → the 4 becomes a 5 (1 more), as the digit to its right is 8, so more than 5

1.3.2 Rounding to a number of decimal places

A **decimal place** is the position of a digit to the right of the decimal point. To round to a number of decimal places, round to the place value located that **many places after the decimal point**.

Example: round 15.485 to 2 decimal places

T	O	.	t	h	th
1	5	.	4	8	5
↓ same	↓ same		↓ same	↓ 1+	↓ zero
1	5	.	4	9	0
				↑	



1.3.3 Rounding to a number of significant figures



A **significant figure** is each of the digits of a number used to express it, from the **first to the last non-zero digit**. To round to a number of significant figures, round to the place value of the last significant figure, counting from left to right from the first non-zero digit.

Example: round 15.485 to 2 significant figures

The first non-zero digit is in the Tens column. The 2 significant figures are in Tens and Ones, so we round to the nearest One.

T	O	.	t	h	th
1	5	.	4	8	5
↓	↓		↓	↓	↓
1	5	.	0	0	0
	↑				

1.3.4 Rounding errors and bounds



Looking at the rounded value of a number, we do not know what the actual value of the number is, but we know which minimum value (**lower bound**) and maximum value (**upper bound**) the number can be. The interval between the lower bound and the upper bound is called the **error interval**.

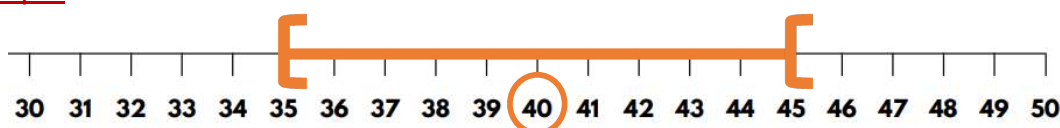
ROUNDING ERRORS, LOWER BOUND, UPPER BOUND, ERROR INTERVAL

Consider the unit (place value) the number is rounded to.

- **Lower bound** = rounded value $- \frac{1}{2}$ unit
- **Upper bound** = rounded value $+ \frac{1}{2}$ unit
- **Error interval** = lower bound \leq number $<$ upper bound

N.B. The lower bound is included in the interval (\leq), but the upper bound is not ($<$)

Example: a number rounded to the nearest 10 is 40



We are rounding to the nearest 10, so our unit is 10 and therefore $\frac{1}{2}$ unit = 5.

The lower bound is $40 - 5 = 35$, the upper bound is $40 + 5 = 45$ and the error interval = $35 \leq \text{number} < 45$.

H

CALCULATING USING ROUNDED NUMBERS

1. Find the lower bound and upper bound of all the numbers used in the calculation
2. Think about how to get the lowest value and the highest result for this calculation

A multiplication is

- a. lowest if each factor is lowest \rightarrow use lower bounds
- b. highest if each factor is highest \rightarrow use upper bounds

A division is

- a. lowest if numerator is lowest (lower bound) and denominator highest (upper bound)
- b. highest if numerator is highest (upper bound) and denominator lowest (lower bound)

Example: a wall measures 2.7m high and 4.4m wide, both measures are given to 1 decimal place.

What is the maximum area of the wall? We know that Area = Height x Width.

As it is a multiplication, its highest value is when each factor is highest.

Our unit is 0.1 as the measures are to 1 decimal place, so we have:

- Height Upper Bound = $2.7 + 0.05 \text{ m} = 2.75\text{m}$
- Width Upper Bound = $4.4 + 0.05 \text{ m} = 4.45\text{m}$
- Maximum wall area = Height UB x Width UB = $2.75 * 4.45 = 12.2\text{m}^2$

1.3.5 Estimating

Estimating is like guessing. Your answer is not accurate, but it is near enough to get a rough idea.

ESTIMATING CALCULATIONS

1. For **whole numbers and decimals**, round each number in the calculation to 1 significant figure
2. For **square roots**, replace the number under the root by its nearer square number
2. Do the calculations respecting BIDMAS, trying to find the easiest order and showing each step

H

Example:

$$\frac{32.5 * \sqrt{27}}{0.481} \approx \frac{30 * \sqrt{25}}{0.5} = \frac{30 * 5}{0.5} = \frac{150}{0.5} = 300$$

1.4 Standard form

1.4.1 Expressing numbers as ordinary numbers and in standard form




We use the standard form to write numbers that are either large or small. It looks like this: $a * 10^n$

- a is a number with **only its first significant figure before the decimal point**. Other significant figures are written after the decimal point. Therefore $1 \leq a < 10$
- n is the power of 10. It represents how far the decimal point moves.
- n is an integer, i.e.
 - **positive whole number** for big numbers (≥ 10),
 - **negative whole number** for small numbers (< 1),
 - **zero** for numbers between 1 and 10


CONVERTING FROM ORDINARY NUMBERS TO STANDARD FORM

1. Work out the value of a
 - Find the first significant figure, write it under the matching digit in the original number
 - Write the decimal point
 - Write the other significant figures after the decimal point
2. Work out if n is
 - **positive**: the number is **big** ≥ 10
 - **negative**: the number is **small** < 1
 - **zero**: the number is **between 1 and 10**
3. Work out the **value of n** = how many places the decimal point moves.

Example: 0.0000579

1. $a = 5.79$
2. n is negative as the number is small and <1
3.  0.0000579 the decimal point moves 5 places, so $n = -5$
 $0.0000579 = 5.79 * 10^{-5}$


Example: 6,400,000,000

1. $a = 6.4$
2. n is positive as the number is big and ≥ 10
3.  6,400,000,000 the decimal point moves 9 places, so $n = +9$
 $6,400,000,000 = 6.4 * 10^9$

CONVERTING FROM STANDARD FORM TO ORDINARY NUMBER

1. Look at the value of n and work out the size of the ordinary number:
 - n is positive: the number is big and ≥ 10
 - n is negative: the number is small and <1
 - n is zero: the ordinary number is between 1 and 10
2. Starting from the decimal point in a , move the decimal point the number of places shown by n , and in the direction to make the number big/small/between 1 and 10, as required.
3. Fill in the empty place values with zeros, up to the 'ones,' if needed.

Example: $2.4 * 10^{-4}$

1. n is negative so the number is small and <1
2. $n = -4$ so we move the decimal point 4 places. We move it to the left to make a small number.
 $a = .\overset{2.4}{}$
3.  0.00024
 $2.4 * 10^{-4} = 0.00024$

Example: $1.25 * 10^2$

1. n is positive so the number is big and ≥ 10
2. $n = 2$ so we move the decimal point 2 places. We move it to the right to make a big number.
 $a = 1.\overset{25}{}$
3. 125
 $1.25 * 10^2 = 125$

1.4.2 Adding and subtracting numbers in standard form

ADDING AND SUBTRACTING NUMBERS IN STANDARD FORM

To add/subtract numbers in standard form, they need to be expressed in the same power of 10.

1. Check all numbers in the same power of 10.
Otherwise, express them in the same power (TIP: it is easier to use the smallest power)
2. Add or subtract the front numbers, and multiply by the common power of ten
3. Convert to standard form if needed.

H

Example: $(3.6 * 10^3) + (1.25 * 10^4)$

1. Convert both numbers so the power of ten is 10^3 : $3.6 * 10^3$ is already done
and $1.25 * 10^4 = 1.25 * 10 * 10^3 = 12.5 * 10^3$
2. Add the front numbers $3.6 * 10^3 + 12.5 * 10^3 = (3.6 + 12.5) * 10^3 = 16.1 * 10^3$
3. $16.1 * 10^3 = 1.61 * 10^4$

1.4.3 Multiplying numbers in standard form

MULTIPLYING NUMBERS IN STANDARD FORM

1. As multiplication is commutative, we can rearrange the order in which we do the multiplication:
 - Place the two **a** at the front and work out their product
 - Place the two 10^n at the back and work out their product using power rules
2. Combine the two answers, and express the result in standard form

Example: $(8.4 * 10^{-4}) * (1.25 * 10^2)$

1. $8.4 * 1.25 * 10^{-4} * 10^2$
 - a. $8.4 * 1.25 = 10.5$
 - b. $10^{-4} * 10^2 = 10^{-2}$
2. $10.5 * 10^{-2} = 1.05 * 10^{-1}$

1.4.4 Dividing numbers in standard form

DIVIDING NUMBERS IN STANDARD FORM

1. Write the division as a fraction
2. Complete the division:
 - Divide the two front numbers
 - Divide the two powers of ten, using power rules
3. Combine the two answers, and express the result in standard form

Example: $(8.4 * 10^{-4}) \div (1.2 * 10^2)$

1. $\frac{8.4 * 10^{-4}}{1.2 * 10^2}$
2. $8.4 \div 1.2 = 7$
 $10^{-4} \div 10^2 = 10^{-6}$
3. $7 * 10^{-6} \rightarrow$ this is already in standard form

1.5 Negative numbers

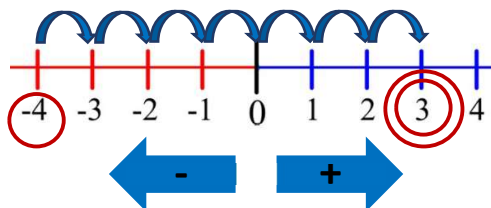
1.5.1 Adding or taking away negative numbers

ADDING OR TAKING AWAY NEGATIVE NUMBERS

1. Draw your number line and locate the first number in the sum on the number line
2. For adding the second number, move up the number of 'steps' (to the right)
For taking away the second number, move down the number of 'steps' (to the left)
3. You land on the answer

Example: $-4 + 7 = ?$

1. Locate -4 on the number line
2. Go up 7 steps
3. You land on 3, which is the answer

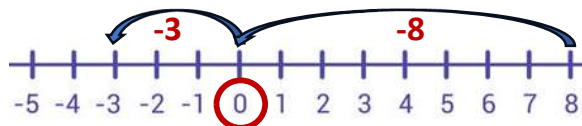


TIP 1: swap the order if easier (+ and - are commutative i.e. the order does not matter) e.g. $-7 + 8 = + 8 - 7 = +1$

TIP 2: 'bridge' via 0 i.e. break down the calculation by stepping on zero, when needed.

Example: $8 - 11 = ?$

First do $8 - 8 = 0$ (land on zero). As you only took away 8, take away another 3, from 0, you land on -3.



1.5.2 Multiplying or dividing negative numbers

MULTIPLYING OR DIVIDING NEGATIVE NUMBERS

1. First solve the sign, using the rule for combining signs in **multiplication and division** (see table on the right)
+ + makes +
- - makes +
+ - makes -
- + makes -
2. Multiply or divide the numbers
3. Combine the sign and number as answer

+ add - take away	+ money - a debt	=	+ richer - poorer
+ add	+ money	=	+ richer
- take away	+ money	=	- poorer
+ add	- a debt	=	- poorer
- take away	- a debt	=	+ richer

Example: $- 2 \times 3 = ?$

1. Apply the rule for combining signs: $- + = -$
2. Multiply the numbers together: $2 \times 3 = 6$
3. Combine sign and number as the answer: **-6**

Example: $-8 \div -2 = ?$

1. Apply the rule for combining signs: $- - = +$
2. Divide the numbers together: $8 \div 2 = 4$

Combine sign and number as the answer: **+4**

TWO SIGNS NEXT TO EACH OTHER

If two signs are next to each other, they **multiply each other**, apply the rule for combining signs in multiplication and division

Example: $- - 2 = ?$

Apply the rule for combining signs:

$$- - = +$$

Apply the sign to the number: **+2**

2 4 operations

2.1 Defining the 4 operations

Although you are probably familiar with the concepts of addition, subtraction, multiplication and division, we revisit what these mean, as well as their key properties and relationships with one another.

2.1.1 Definitions



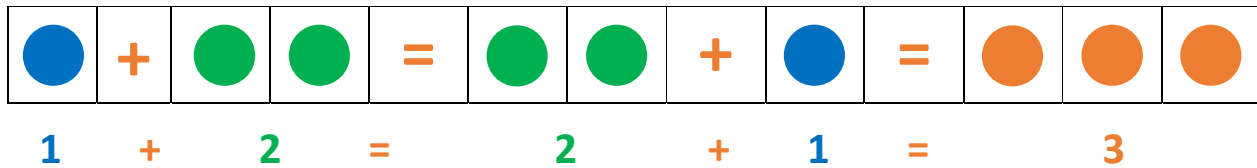
Addition +	Subtraction -
<p>Addition is the sum of 2 numbers. It is when 2 numbers are combined to get a new total.</p> <p>The operator is + and the result is the sum.</p> <p>e.g. My flatmate bought 5 bananas and I bought 4 bananas. Altogether we now have 9 bananas.</p> <p style="text-align: center;">5 + 4 = 9</p> <p style="text-align: center;">This is the sum of 5 bananas and 4 bananas, combined together, there are 9 bananas now</p> <p>Keywords: add, altogether, and, combined, increase, make, more, plus, sum, total.</p>	<p>Subtraction is when we take a number away from another number.</p> <p>The operator is - and the result is the difference.</p> <p>e.g. My son saved £10 of pocket money. He used £2 to buy a candy. He now has £8 left.</p> <p style="text-align: center;">10 - 2 = 8</p> <p style="text-align: center;">He took away £2 from his £10: he is left with £8</p> <p>Keywords: difference, fewer, left, less, minus, reduce, remain, subtract, take away, take out</p>
Multiplication	Division
<p>Multiplication is a repeated addition of a number, it is how many times a number is added to itself.</p> <p>The operator is x and the result is the product.</p> <p>e.g. I have 3 packs of 6 sodas. I have 18 altogether.</p> <p style="text-align: center;">3 x 6 = 18</p> <p style="text-align: center;">3 lots of 6 sodas is 6 + 6 + 6 = 18 sodas</p> <p>Keywords: product, multiply, times, by, groups of, lots of, packs of, etc.</p>	<p>Division is splitting or sharing an amount in equal-size groups.</p> <p>The operator is ÷ and the result is the quotient.</p> <p>e.g. 2 friends share 6 sweets, they get 3 each.</p> <p style="text-align: center;">6 ÷ 2 = 3</p> <p>Keywords: quotient, divide, share, split, break.</p>

Note: we can only add or subtract the **same type of things** e.g. you cannot add 5 bananas and 4 apples, but you could add their price as these are then in the same unit (same type of thing) i.e. £.

2.1.2 Properties of operations

Addition

- **Associative:** $(a + b) + c = a + (b + c) \rightarrow$ we can group the operations in any combination, i.e. the order in which we add numbers does not matter, they are combined together to the same total
For example: $(3 + 1) + 2 = 3 + (1 + 2) \rightarrow 4 + 2 = 3 + 3 = 6$
- **Identity property:** $0 + a = a + 0 = a \rightarrow$ adding 0 to a number does not change this number, i.e. we are adding nothing, so the number is unchanged
For example: $0 + 7 = 7 + 0 = 7$
- **Additive inverse:** $a + -a = -a + a = 0 \rightarrow$ adding two identical numbers of opposite sign equals 0, i.e. if you go up a number of steps and then go down the same number of steps, you are back at 0
For example: $5 + -5 = -5 + 5 = 0$
- **Commutative:** $a + b = b + a \rightarrow$ same result regardless of the order you carry out the operations, i.e. the numbers are combined together into a total, whatever the order in which we count them
For example: $6 + 4 = 4 + 6 = 10$

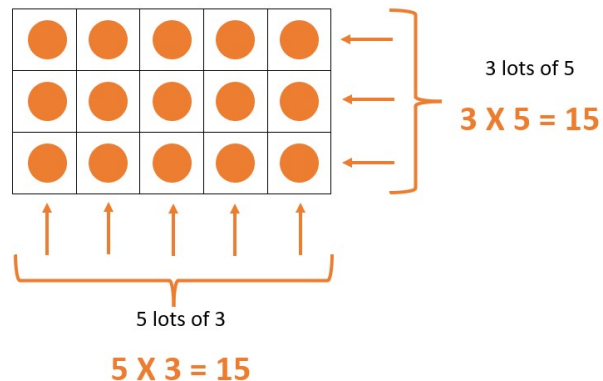


Subtraction

- **Identity property:** $a - 0 = a \rightarrow$ subtracting 0 from a number does not change this number, i.e. we are taking away nothing, so the number is unchanged
For example: $12 - 0 = 12$
- **Equality property:** $a - a = 0 \rightarrow$ subtracting a number from itself equals 0, i.e. we are taking away as many as there are in the number, so we are left with nothing
For example: $15 - 15 = 0$

Multiplication is

- **Associative:** $(a \times b) \times c = a \times (b \times c) \rightarrow$ we can group the operation in any combination, i.e. the order in which we multiply numbers does not matter, they multiply together to the same total
For example: $(3 \times 4) \times 2 = 3 \times (4 \times 2) \rightarrow 12 \times 2 = 3 \times 8 = 24$
- **Identity property:** $1 \times a = a \times 1 = a \rightarrow$ multiplying a number by 1 equals this number, i.e. we are taking the number 1 time so it is just itself
For example: $1 \times 4 = 4 \times 1 = 4$
- **Zero property:** $0 \times a = a \times 0 = 0 \rightarrow$ multiplying any number by 0 equals 0, i.e. taking 0 time a number equals 0
For example: $0 \times 12 = 12 \times 0 = 0$
- **Commutative:** $a \times b = b \times a \rightarrow$ multiplying two numbers gives the same result regardless of the order you multiply them, as shown on the diagram
For example: $3 \times 5 = 5 \times 3 = 15$



Division

- **Identity property:** $a \div 1 = a$ → dividing a number by 1 equals the number, i.e. we divide the number into 1 group
For example: $24 \div 1 = 24$
- **Zero property 1:** $a \div 0 =$ impossible → dividing by 0 is not possible
- **Zero property 2:** $0 \div a = 0$ → 0 divided by any number equals 0, i.e. we divide nothing in equal parts, we still have nothing
For example: $0 \div 4 = 0$
- **Dividing by itself:** $a \div a = 1$ → any number divided by itself equals 1, e.g. we divide the number in as many groups as the number, so we place 1 in each group
For example: $9 \div 9 = 1$

2.1.3 Inverse operations

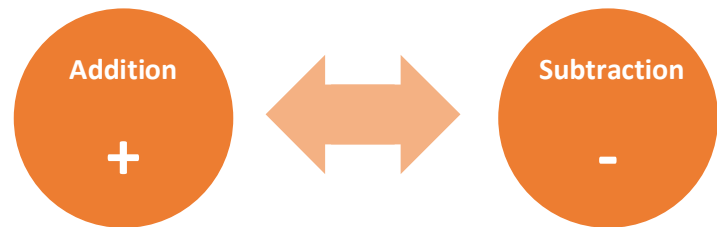


An **inverse operation** reverses the effect of another operation. Inverse operations are opposites of each other, they undo each other.

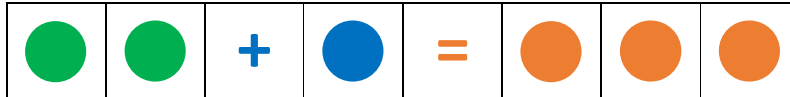
Addition and subtraction

Addition and subtraction are **inverse operations**.

Example: $2 + 1 = 3$ and $3 - 1 = 2$

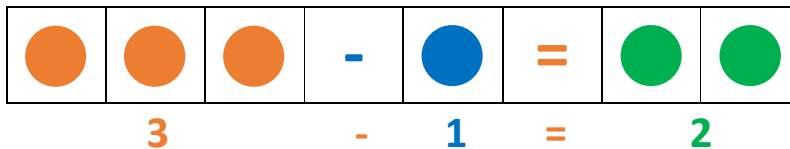


First, we add 1 to 2, which makes 3.



$$2 + 1 = 3$$

Then, we subtract 1 from 3, and we are back at 2.

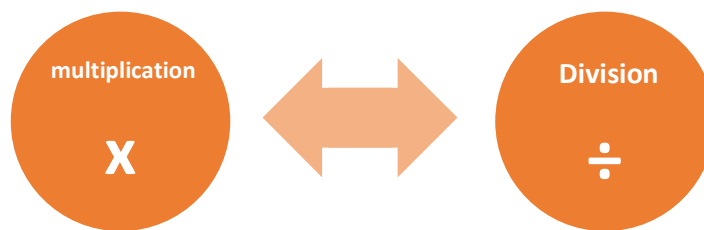


$$3 - 1 = 2$$

Subtracting 1 has cancelled out the 1 we had added. Subtraction does the opposite of what addition does. When we add and then subtract the same number, we get the number we started with.

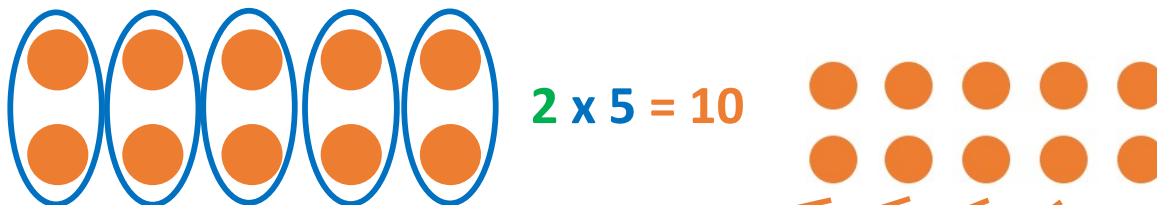
Multiplication and division

Multiplication and division are **inverse operations**.

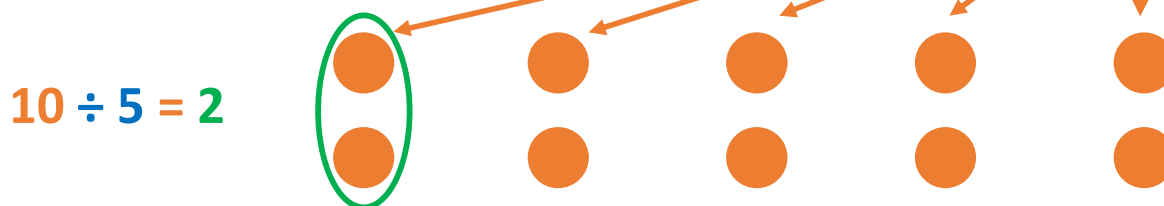


Example: $3 \times 5 = 15$ and $15 \div 5 = 3$

First, we multiply the number 2 5 times.



Then, we divide 10 by 5, and we are back at 2.



Dividing by 5 has cancelled out the multiplying by 5. Division does the opposite of what multiplication does. When we multiply and then divide by the same number, we get the number we started with.

2.1.4 BIDMAS – in which order to carry out operations



BIDMAS

Brackets **()**

Indices $5^2 \sqrt{9}$

Division \div

Multiplication **X**

Addition **+**

Subtraction **-**

In fact:

\div and **X** have same priority \rightarrow apply left to right

+ and **-** have same priority \rightarrow apply left to right

Example:

$(3 + 3 \times 2) + 18 \div 6 \rightarrow$ first bracket & BIDMAS inside

$= (3 + 6) + 18 \div 6 \rightarrow$ solve the bracket

$= 9 + 18 \div 6 \rightarrow$ division before addition

$= 9 + 3 = 12 \rightarrow$ addition

B ()

I $5^2 \sqrt{9}$

DM $\div \times$

AS $+-$

2.2 Adding whole numbers and decimals

ADDING NUMBERS (WHOLE OR DECIMALS) USING A COLUMN ADDITION

1. Write the numbers to be added one under the other and draw a line. Use a place value frame or make sure that you line up the place values (TIP: line up the 'ones' or 'decimal points' first).
2. Add the digits in the column on the right. If the result is above 10, write down the 'ones' digit underneath and carry over the 'tens' to the next column to the left.
3. Repeat as above for the next column to the left, do not forget to add the number carried over.
4. Keep going until you have added the numbers in the left column.

	T TENS	O ONES	t tenths	h hundredths
	2	6	3	5
+	3	4	9	3
	¹ 6	¹ 1	2	8

Example: $26.35+34.93=?$

1. Write both numbers in a place value frame, one under the other and draw a line.
2. Adding the numbers in the right column: $5+3 = 8$. Write 8 underneath.
3. Move to the next column: $3+9 = 12$. Write 2 underneath and carry over the 1. Move to the next column: $6+4+1 = 11$. Write 1 and carry over the 1.
4. Move to the last column: $2+3+1=6$. The answer is 61.28.

2.3 Subtracting whole numbers and decimals

SUBTRACTING NUMBERS (WHOLE OR DECIMALS) USING A COLUMN SUBTRACTION

1. Write the number you are subtracting from at the top and the number you are subtracting underneath. As above, use a place value frame or make sure that you line up the place values.
2. Starting from the column on the right, try to subtract the bottom digit from the top digit.
 - If it is possible, write the answer underneath.
 - If it is not, 'borrow' 1 from the digit to the left of the top digit → this 1 is worth 10 for the top digit, as it is from one place value above. Now subtract the bottom digit from the top digit.
3. Repeat as above for the next column to the left, borrowing when needed.
4. Keep going until you have subtracted the numbers in the left column.

	T TENS	O ONES	t tenths	h hundredths
	3 2	1 4	9 8	1 3
-	2	6	3	5
	0	8	5	8

Example: $34.93-26.35=?$

1. Write 34.93 at the top and 26.35 underneath. Draw a line.
2. Starting from the column on the right, try 3-5. It is not possible, borrow 1 from 9 (which becomes 8), add it to the 3, its value is now 10, making it 13. Now you can do $13-5=8$. Write 8 underneath.
3. Move to the next column: $8-3 = 5$. Next, $4-6$ is not possible. Borrow 1 from the 3 (which becomes 2) and add it to the 4. Its value is 10, so $14-6=8$.
4. Move to the last column: $2-2=0$. Answer = 8.58.

2.4 Multiplying whole numbers and decimals


2.4.1 Multiplying by 10, 100, 1,000 etc.

MULTIPLY BY 10, 100, 1000 ETC.


1. LOCATE your **decimal point** (if whole number, add it at the end, after the 'ones')
2. COUNT the **number of zeros** in the multiplying number
 - MOVE the **decimal point**: you are **MULTIPLYING** so the number is getting **bigger** → move the decimal point to the **RIGHT** as many places as the number of zeros in the multiplying number
3. FILL the **empty place values** with zeros as necessary i.e. a
 - For numbers >1 , all place values in the whole number part are filled
 - For numbers <1 , all place values from 'ones' to smallest in fractional part are filled

Examples of multiplications: 23×100 and $432.6 \times 1,000$

23×100

23. 1. Locate your decimal point (and add it after the ones, as 23 is a whole number)
- 100 2. Count the number of zeros in the multiplying / dividing number i.e. 2 zeros here
- 23  . 3. Move the decimal point 2 times to the right (2 zeros in 100 and we are multiplying)
- 2300 4. Fill the number with zeros as necessary

$432.6 \times 1,000$

- 432.6 1. locate your decimal point
- 1000 2. Count the number of zeros in the multiplying / dividing number i.e. 3
- 432.6  . 3. Move the decimal point 3 times to the right (3 zeros in 1000 and we are multiplying)
- 432,600 4. Fill the empty place values with zeros as necessary

2.4.2 Multiplying by 20, 300, 6,000 etc.

MULTIPLY BY 20, 300, 6000 ETC.

1. Multiply by the **whole number** first
2. Multiply the result by **10, 100, 1,000** etc.

Example: $\times 2,000 \equiv \times 2 \times 1,000$

$234 \times 2,000$

$234 \times 2 = 468$ 1. Multiply by whole number

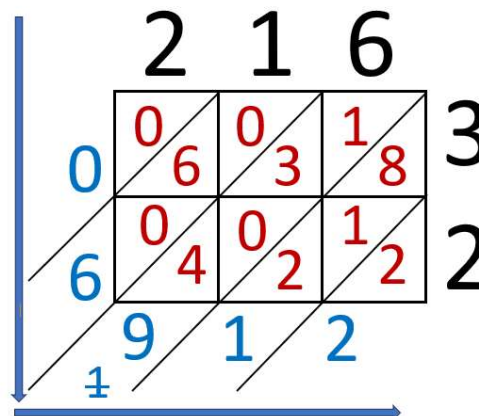
$468 \times 1,000 = 468,000$ 2. Multiply result by 1,000

2.4.3 Multiplying whole numbers and decimals using the lattice method

Example: 216 x 32**LATICE MULTIPLICATION OF WHOLE NUMBERS**

1. Draw the grid (**one digit per column/row**) and align the numbers (in black)
2. For each rectangle, **multiply** the digit at the top by the one to the right, write the answer in that rectangle, with 'tens' and 'ones' separated by the diagonal (in red) e.g. $6 \times 3 = 18$
3. Add the numbers **in each diagonal**, start on the right and write total at the bottom (in green) N.B **Carry over** the 'tens' number when the total is a 2-digit number (like in column addition)
4. The **result** is the number underneath the frame, read from top left to bottom right

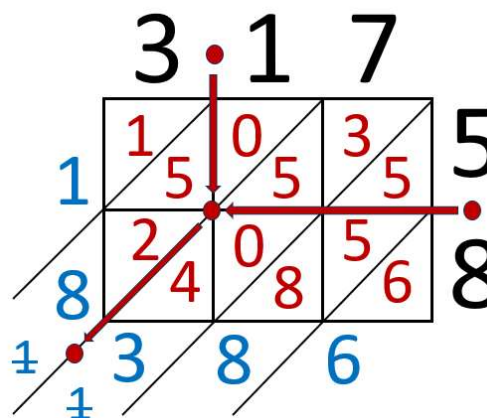
1. 216 at the top and 32 on the right, 1 digit per column
2. $6 \times 3 = 18$, 1 above diagonal and 8 under diagonal etc.
3. In 2nd diagonal column from the right: $8 + 1 + 2 = 11 \rightarrow$
I write 1 underneath column and carry 1 to next column
4. Total read from top right to bottom left = 6,912

**LATICE MULTIPLICATION OF DECIMAL NUMBERS**

1. **Slide both decimal points** following the vertical and horizontal lines until they meet
2. Then slide the resulting point **along the diagonal line** to the bottom. This is where the decimal point will be in the answer.
3. Carry out multiplication as per method 4

Example: 3.17 x 5.8

1. Slide the decimal point from 3.17 down one and the decimal point from 5.8 to the left until they meet
2. Slide the resulting decimal point down the diagonal line until underneath the frame
3. Carry out multiplication as per method above



2.4.4 Multiplying whole numbers and decimals using the column method

STANDARD COLUMN MULTIPLICATION OF WHOLE NUMBERS

- Write the numbers you are multiplying one under the other, respecting the place values (**one digit per place value column**) (in black) and draw a line under it.
TIP: write the number with the less digits at the bottom.
- One by one, multiply each digit from the bottom number to the entire top number and write the answer on a separate line each time, starting with the digit in the 'ones' – see example below. You might have to carry over the 'tens' number if any answer is 10 or above.
- Repeat for each digit in the bottom number.
- Draw a line and add up the numbers you have just worked out above, the total is the answer!
N.B **Carry over** the 'tens' digit when the total is a 2-digit number (like in column addition)

Example: 416 x 25

- 416 at the top and 25 at the bottom
- First do 5 x 416** (5 in the 'ones' of bottom number) – (3 blue arrows)
 - $5 \times 6 = 30$, write 0 in the 'ones' (6 is in the 'ones') and carry over the 3 to the next column (write it small & under in the 'tens')
 - $5 \times 1 = 5$ then do + 3 that was carried over = 8. Write it in the 'tens' (1 is in the 'tens')
 - $5 \times 4 = 20$, write the 0 in the 'hundred' (4 is in the 'hundreds') and carry over the 2 to the next column (write it in the 'thousands' as they are no more multiplication on this line.)
- Next do 20 x 416** As $20 = 10 \times 2$, first, multiply by 10 by adding a 0 in the 'ones' then multiply by 2. Write the answer from the **'tens'** not the 'ones' – purple arrows
 - $2 \times 6 = 12$, write 2 in the 'tens', carry over 1 to the 'hundreds'
 - $2 \times 1 = 2$ then do + 1 = 3. Write it in the 'hundreds'
 - $2 \times 4 = 8$, write 8 in the 'thousands'
- Draw a line and add up 2,080 and 8,320 in a column addition.

TTh	Th	H	T	O	
		4	1	6	
					x
			2	5	
	2	0	8	0	← 5x416
	8	3	2	0	← 20x416
1	0	4	0	0	

STANDARD COLUMN MULTIPLICATION OF DECIMAL NUMBERS

- Multiply each number **by 10, 100, 1,000** etc. until the numbers are whole numbers
- Carry out the multiplication between the whole numbers (as per method above)
- Divide the answer **by the same 10, 100, 1,000** you used to make the numbers whole

Example: 4.16 x 2.5

- Multiply 4.16 **by 100** to get 416,
Multiply 2.5 **by 10** to get 25
- Multiply the 416 x 25 as above
- Divide the solution **by 100**, and then **by 10**

4.16	x	2.5	=	10.4
↓ x 100		↓ x 10		↑ ÷ 10
416	x	25	=	104
				↑ ÷ 100
416	x	25	=	10,400

2.5 Dividing whole numbers and decimals

2.5.1 Dividing by 10, 100, 1,000 etc.

DIVIDING BY 10, 100, 1000 ETC.


1. LOCATE your **decimal point** (if whole number, add it at the end, after the 'ones')
2. COUNT the **number of zeros** in the dividing number
3. MOVE the **decimal point**: you are **DIVIDING** so the number is getting **smaller** → move the decimal point to the **LEFT** ← as many places as the number of zeros in the dividing number
4. FILL the **empty place values** with zeros as necessary i.e. a
 - For numbers >1, all place values in the whole number part are filled
 - For numbers <1, all place values from 'ones' to smallest in fractional part are filled

Examples: $84 \div 100$ and $2.31 \div 1,000$

$$84 \div 100$$

84. 1. Locate your decimal point

100 2. Count the number of noughts in the multiplying / dividing number i.e. 2


 **.84.** 3. Move the decimal point 2 times (as there are 2 noughts in 100)

0.84 4. Fill the number with zeros as necessary (before and after)

$$2.31 \div 1000$$

2.31 1. locate your decimal point

1000 2. Count the number of noughts in the multiplying / dividing number i.e. 3

 **. 2.31** 3. Move the decimal point 3 times (as there are 3 noughts in 1000)

0.00231 4. Fill the number with zeros as necessary (before and after)

2.5.2 Dividing by 20, 300, 6,000 etc.

DIVIDING BY 20, 300, 6000 ETC.

1. Divide by **10, 100, 1,000** etc. first
2. Divide the result by **whole number**

Example: $\div 300 \equiv \div 100 \div 3$

$$66,000 \div 300$$

$66,000 \div 100 = 660$ 1. Divide by 100

$660 \div 3 = 220$ 2. Divide result by whole number

2.5.3 Dividing a whole number by a whole number

BUS METHOD FOR DIVIDING A WHOLE NUMBER BY ANOTHER WHOLE NUMBER

1. Write the number you are dividing under a 'bus,' with a line going over and to its left. Write the dividing number to the left of the line.
2. Look at how many times the number on the left goes into the one on the right, starting with the highest place value digit. Write the answer above this digit and carry over the remainder to the next digit on the right. If it does not go into it, write 0 and carry the digit over (as remainder).
3. Repeat until you reach the last digit of the number you are dividing.
4. If there is a remainder for the last digit, you can:
 - give the answer with remainder: add r after the solution and the value of the remainder
 - try to find the exact solution: add a zero after the last digit of the number you are dividing (add a decimal point if needed) and carry over the remainder. Repeat.

$182 \div 7$

Example: $182 \div 7$

1. Write 182 under the bus and 7 to the left.
2. How many times does 7 goes into 1 (hundred column)? 7 can't go into 1, write 0 at the top and carry 1 over to the right → now 18
3. 7 can go 2 times into 18, with 4 left-over, so write 2 at the top and carry 4 over to the right → now 42
- 7 can go 6 time into 42, write 6 at the top.

$$\begin{array}{r}
 0 \quad 2 \quad 6 \\
 7 \overline{) 182} \\
 \underline{18} \\
 42 \\
 \underline{42} \\
 0
 \end{array}$$

$2 \times 7 = 14$
 $18 - 14 = 4$

$6 \times 7 = 42$

2.5.4 Dividing a decimal by a whole number

BUS METHOD FOR DIVIDING A DECIMAL NUMBER BY A WHOLE NUMBER

1. As above, but add a decimal point exactly above the one in the number you are dividing

Example: $18.2 \div 7$

Add a decimal point above the one in 18.2, do the division as per method above.

$$\begin{array}{r}
 0 \quad 2 \quad . \quad 6 \\
 7 \overline{) 18.2} \\
 \underline{18} \\
 02 \\
 \underline{02} \\
 0
 \end{array}$$

2.5.5 Dividing any number by a decimal

DIVIDING A NUMBER BY A DECIMAL

1. Write the division as a fraction
2. Multiply the denominator by 10, 100, 1000 etc. so it is a whole number
3. Multiply the numerator by the same as the denominator, so it is an equivalent fraction
4. Apply division method above

$$14.2 \div 0.4 = \frac{14.2}{0.4} = \frac{142}{4}$$

X10
↻
↻
X10

Example: $14.2 \div 0.4$

1. Write it as a fraction: $\frac{14.2}{0.4}$
2. Multiply the denominator by 10, so it becomes a whole number i.e. 4
3. Multiply the numerator by 10, so the new fraction is equivalent
4. Solve the division using the bus method: $142 \div 4 = 35.5$

3 Primes, factors, multiples, prime factors, LCM and HCF

3.1 Primes



- A **prime number** is a number that has two factors: 1 and itself. The number = $1 \times \text{itself}$
 - The only numbers that multiply to give 7 are 1 and 7 ($7 = 1 \times 7$) → 7 is a prime
 - The only numbers that multiply to give 31 are 1 and 31 ($31 = 1 \times 31$) → 31 is a prime

Some facts:

- 1 is not a prime
- 2 is the only even number that is a prime
- 5 is the only number ending in 5 that is a prime
- 2, 3, 5 and 7 are the only prime numbers under 10
- From 10 onwards, all prime numbers end in 1, 3, 7 or 9

IS IT A PRIME?

It is a prime if you answer **NO** to 3 questions (for numbers above 10)

1. Does the number end in 0, 2, 4, 6, 8, or 5?
2. Can the number divide exactly by 3?
3. Can the number divide exactly by 7?

End in 0,2,4,6,8,5 ✖
Divide exactly by 3
Divide exactly by 7

Examples:

Is 53 a prime?

1. Does 53 end in 0, 2, 4, 6, 8, or 5? **NO**
 2. Can the number divide exactly by 3? $53 \div 3 = 17$ remainder 2 → **NO**
 3. Can the number divide exactly by 7? $53 \div 7 = 7$ remainder 4 → **NO**
- ⇒ 53 is a prime

Is 77 a prime?

1. Does 77 end in 0, 2, 4, 6, 8, or 5? **NO**
 2. Can the number divide exactly by 3? $77 \div 3 = 25$ remainder 2 → **NO**
 3. Can the number divide exactly by 7? $77 \div 7 = 11$ → **YES**
- ⇒ 77 is not a prime as $77 = 7 \times 11$

3.2 Multiples and factors



- **The multiples of a number** are the numbers that appear in the times table of this number.
 - Multiples of 7 = 7, 14, 21, 28, 35, 42, 49 etc.
 - Multiples of 15 = 15, 30, 45, 60, 75, 90 etc.



- **The factors of a number** are the numbers that divide exactly into it.
 - Factors of 6 = 1, 2, 3, 6
 - Factors of 20 = 1, 2, 4, 5, 10, 20

FINDING THE FACTORS OF A NUMBER

To find all the factors of a number, find all the pairs that multiply into this number.

1. Start at 1 x number itself
2. Increase by 1 each time
3. Cross out the lines where it does not divide exactly
4. When you repeat a number, you can stop as you have found all the factors

Examples:

What are the factors of 12?

$$1 \times 12$$

$$2 \times 6$$

$$3 \times 4$$

~~4~~ × ~~3~~ → repeat of 3 × 4 but swapped, we can stop here as both 3 and 4 appear above

The factors of 12 are **1, 2, 3, 4, 6, 12**.

What are the factors of 25?

$$1 \times 25$$

~~2~~ × → does not divide exactly by 2

~~3~~ × → does not divide exactly by 3

~~4~~ × → does not divide exactly by 4

5 × 5 → Numbers are identical, you can stop.

The factors of 25 are **1, 5, 25**.

3.3 Prime factors



To find the **prime factors** of a number, we break the number down into two factors, and each factor into its own factors, etc. until these are primes and cannot not be broken down any longer. We then write the number as a **product of its prime factors**.

FINDING THE PRIME FACTORS OF A NUMBER USING THE CHERRY TREE

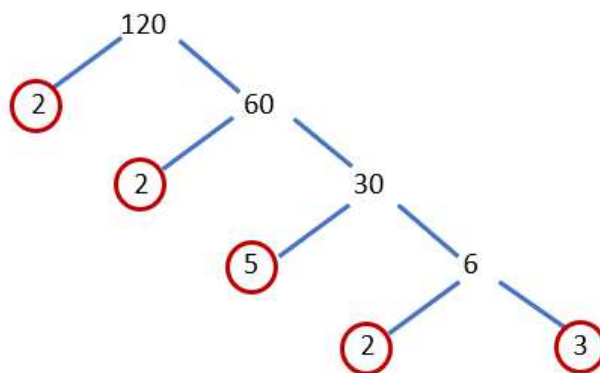
1. Write the number at the top
2. Split it into two factors on the line below, linking back to the original number above
3. If the factors can be split into their own factors, do as above
4. When a number is a prime, circle it
5. When you cannot split further, write the number as product of its prime factors (in order)

Example: Find the prime factors of 120

- Write 120 at the top
- Split 120 into two factors on the line below, i.e. 2 and 60
- Circle 2 as it is a prime factor
- Split 60 into two factors on the line below, i.e. 2 and 30
- Circle 2 as it is a prime factor
- Etc.

120 can be written as a product of its prime factors as

$$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$$



TIP: list the prime factors in order of size, and use powers when a factor multiplies itself twice or more.

3.4 Lowest Common Multiple (LCM)



The **lowest common multiple of a set of numbers** is the smallest number that is a multiple of all these numbers.

LCM – LISTING

1. List the multiples of all the numbers you wish to find the lowest common multiple of
2. Find the smallest number that appear in all the lists of multiples

Example: What is the lowest common multiple of 12, 15 and 20?

1. List the multiples for each number:
 - **12** → 12, 24, 36, 48, **60**, 72, 84, 96, 108
 - **15** → 15, 30, 45, **60**, 75, 90, 105, 120
 - **20** → 20, 40, **60**, 80, 100, 120
2. 60 is the smallest number that is in all the lists of multiples, 60 is the LCM of 12, 15 and 20

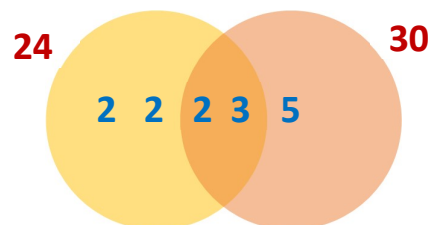
This method works well with small numbers, and it is easy to understand as it shows the multiples. It is not easy to use with big numbers and it is best to use the method below.

LCM – USING PRIME FACTORS IN A VENN DIAGRAM

1. List all the prime factors of each number
2. Create a Venn diagram with a circle for each of the numbers we are looking for the LCM for and place the prime factors accordingly in the Venn diagram, using the following rules:
 - If a prime factor appears more than once in a number, it appears as many times in it
 - If a prime factor is common to two or more numbers, it goes in their intersection
3. The LCM is the product of all the prime factors shown in the Venn diagram

Example: What is the lowest common multiple of 24 and 30?

1. List the prime factors in 24 and 30
 - **24** = $2 \times 2 \times 2 \times 3 = 2^3 \times 3$
 - **30** = $2 \times 3 \times 5$
2. Place the prime factors in a Venn diagram
 - 24 and 30 have 2 and 3 as common factors, they are placed in the intersection
 - The remaining factors for 24 and 30 are placed in their respective circles
3. The LCM is therefore $2 \times 2 \times 2 \times 3 \times 5 = 120$



LCM – Lowest Common Multiple: it is all in the name!

MULTIPLE - to be a multiple of a number, you need to include each number at least, so

- to be a multiple of 24, the LCM needs to include 24 so $2 \times 2 \times 2 \times 3$

- to be a multiple of 30, the LCM needs to include 30 so $2 \times 3 \times 5$

COMMON - to be common, the LCM needs to be a multiple of BOTH 24 AND 30, so to include $2 \times 2 \times 2 \times 3$ and $2 \times 3 \times 5$

LOWEST - to be lowest, it includes as few factors as possible, common factors are not repeated

3.5 Highest Common Factor (HCF)



The **highest common factor of a set of numbers** is the biggest number that divides in all these numbers.

HCF – LISTING

- List all the factors of each number
- Find the biggest number that is in all the lists of factors

Example: What is the highest common factor of 48, 60 and 80?

- List the factors of all the numbers
 - 48** → 1, 2, 3, 4, 6, 8, **12**, 16, 24, 48
 - 60** → 1, 2, 3, 4, 5, 6, 10, **12**, 15, 20, 30, 60
 - 72** → 1, 2, 3, 4, 6, 8, 9, **12**, 18, 24, 36, 72
- 12 is the biggest number in the lists of factors, 12 is the highest common factor of 48, 60 and 72

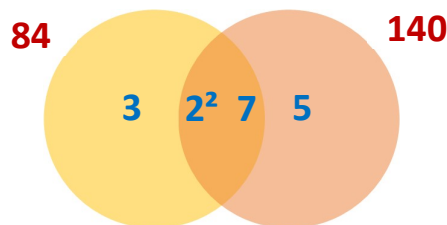
This method works well and is easy to understand as it clearly shows the factors. It is not easy to use with big numbers. Finding factors of big numbers is quite tricky, but you can use the method below.

HCF – USING PRIME FACTORS IN A VENN DIAGRAM

- List all the prime factors of each number
- Create a Venn diagram with a circle for each of the numbers we are looking for the LCM for and place the prime factors accordingly in the Venn diagram, using the following rules:
 - If a prime factor appears more than once in a number, it appears as many times in it
 - If a prime factor is common to two or more numbers, it goes in their intersection
- The HCF is the product of the prime factors common to all numbers, i.e. in the intersection

Example: What is the highest common factor of 84 and 140?

- List the prime factors in 84 and 140
 - 84** = $2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$
 - 140** = $2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
- Place the prime factors in a Venn diagram
 - 84 and 140 have 2^2 and 7 as common factors, they are placed in the intersection
 - The remaining factors for 84 and 140 are placed in their respective circles
- The HCF is therefore $2^2 \times 7 = 28$



HCF – Highest Common Factor: it is all in the name!

COMMON FACTOR – the common factors are the factors that are common to a set of number. We place the common factors in the intersection of the Venn diagram

HIGHEST - to be highest, we use all the common factors and multiply them together

4 Fractions, decimals, and percentages

4.1 Fractions

4.1.1 What is a fraction?

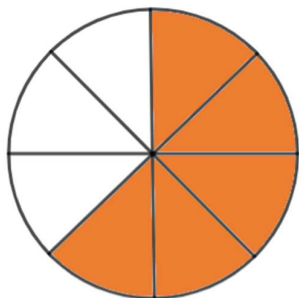


We use **fractions** to look at **parts of a whole**. We spit one whole in **equal size parts**.

Fractions are written as two numbers on top of one another, separated by a **fraction line**.

- The **numerator** is at the top. It represents the **number of parts we are considering**.
- The **denominator** (at the bottom) is the **total number of parts** one whole has been shared into.

Example: This is how we draw and write a fraction of five eighths:



5 ← **numerator:** we are considering the 5 parts coloured in orange

— ← **fraction line** (aka vinculum)

8 ← **denominator:** the whole is split in 8 equal size parts

Fractions are often defined as ‘**parts of a whole**.’ When thinking about fractions, we need to identify what represents the ‘whole’ i.e. what makes the total number of parts in the denominator.

This **fraction circle** is a way to visually represent the fraction of $\frac{5}{8}$. There are many ways to visually represent the same fraction: part of a group, number line, fraction bar, fraction rectangle, etc.

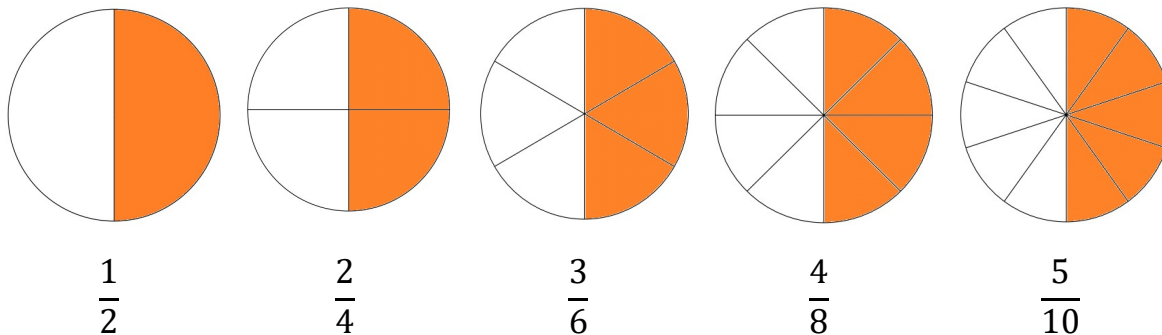
As parts of a group	On a number line	On a fraction bar
<p>Whole = 1 group of 8 identical circles</p>	<p>Whole = total distance from 0 to 1, split in 8 equal parts</p>	<p>Whole = 1 full bar, split in 8 equal parts</p>

4.1.2 Equivalent fractions, simplifying fractions

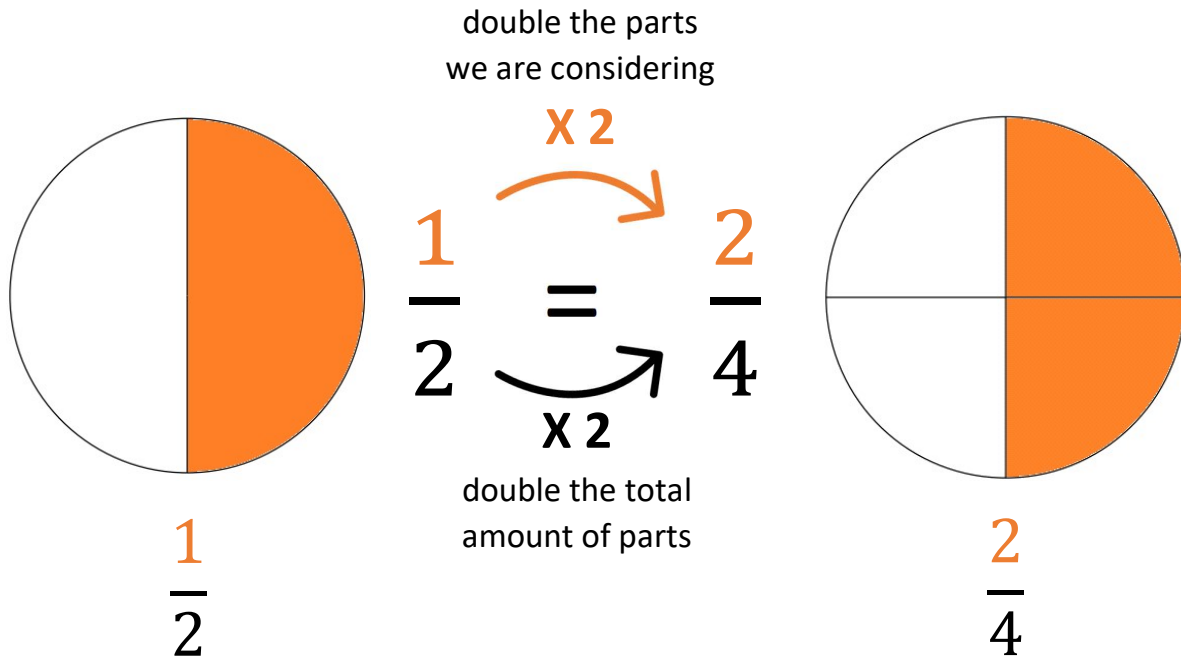


Which amount of pizza would a pizza lover prefer to eat? It doesn't matter, these are the same quantity!

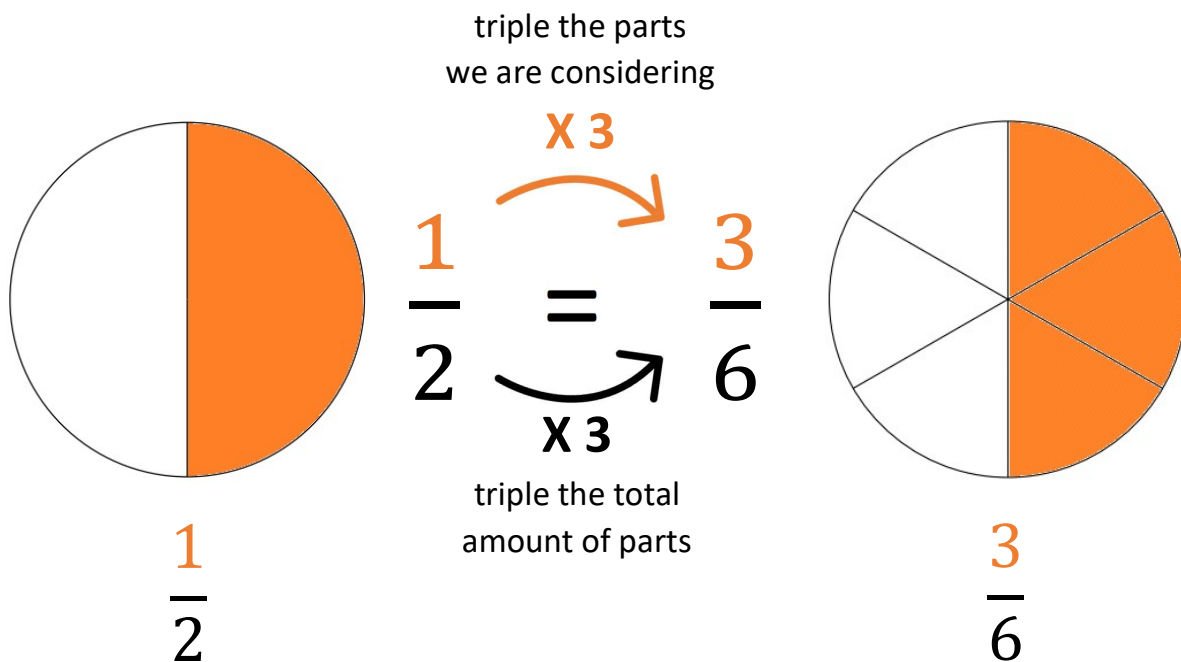
These are called **equivalent fractions**. We can write $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ etc.



From $\frac{1}{2}$ to $\frac{2}{4}$, each part is halved, making double the number of parts, but the parts are half the size.



From $\frac{1}{2}$ to $\frac{3}{6}$, each part is split in 3, so triple the number of parts, but the parts are 3 times smaller.



FINDING EQUIVALENT FRACTIONS

Multiply or divide the numerator and denominator by the **same number** to get an equivalent fraction. **Equivalent fractions represent the same amount**, but with different size parts, and therefore different number of parts.

We use equivalent fractions to **simplify a fraction**. The aim is to find the equivalent fraction in its **simplest form**, which means with the smallest possible numbers.

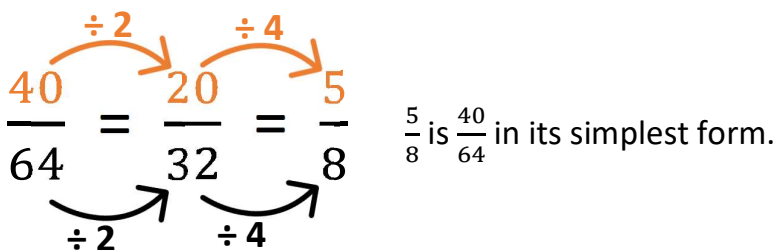
SIMPLIFYING FRACTIONS – METHOD 1: STEP BY STEP

1. Find a common factor to the numerator and denominator. Divide both numerator and denominator by this factor to get an equivalent fraction.
2. Repeat the above until there are no common factors left

SIMPLIFYING FRACTIONS – METHOD 2: CANCELLING OUT COMMON FACTORS

1. Write both numerator and denominator as a product of factors, showing any common factors
2. Cancel out any factor that is both at the numerator and the denominator. This is because a number divided by itself equals 1 and multiplying by 1 does not change the result.

Example: simplify $\frac{40}{64}$ (i.e. write $\frac{40}{64}$ in its simplest form)



$\frac{40}{64} = \frac{\cancel{2} * \cancel{4} * 5}{\cancel{2} * \cancel{4} * 8} = \frac{5}{8}$ is $\frac{40}{64}$ in its simplest form.

A **fraction wall** is an effective way to visualise equivalent fractions and to compare fractions too.

Equivalent fractions:

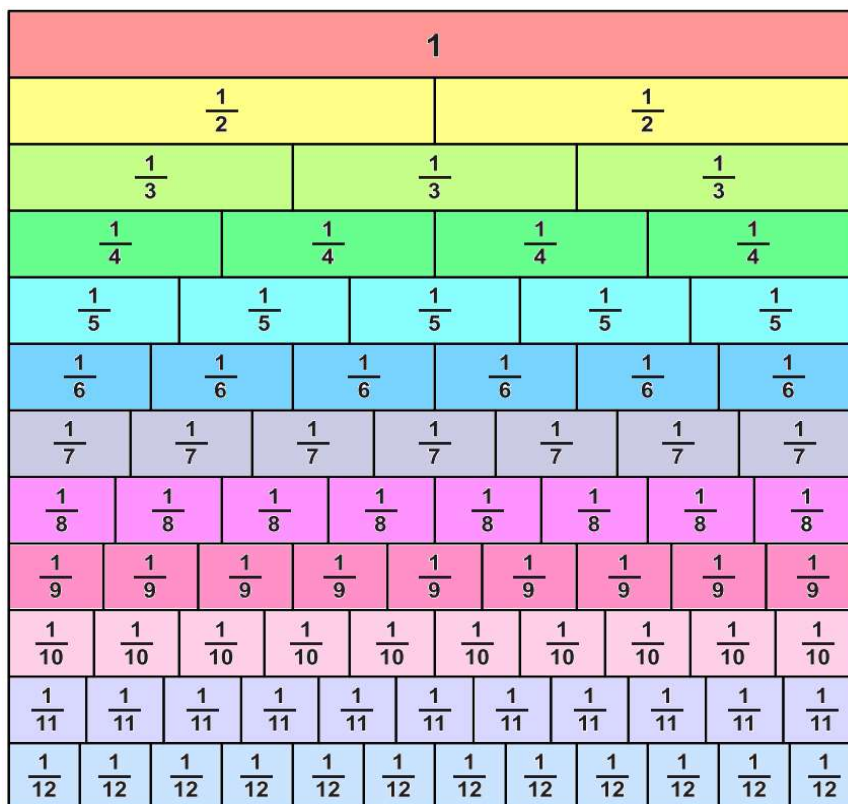
$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

Comparing fractions:

$\frac{5}{7} > \frac{2}{3}$

$\frac{1}{10} < \frac{1}{9}$



4.1.3 Adding and subtracting fractions

We can only add or subtract fractions if they have the **same size parts**. We cannot easily say what $\frac{1}{3} - \frac{1}{4}$ is because thirds and quarters are not the same thing i.e., they are not the same size.

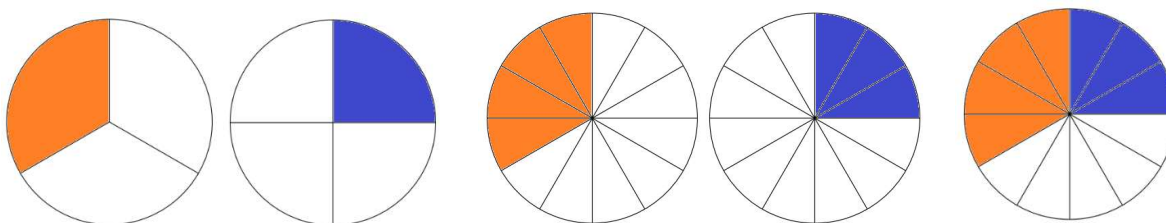
The size of the parts in a fraction is given by its denominator i.e. number of parts a whole is split into.

To add or subtract fractions, we need to find equivalent fractions, so the fractions are **expressed over the same denominator** (in the same size parts!).

ADDING OR SUBTRACTING FRACTIONS

1. If needed, express all fractions over the **same denominator**. TIP: use LCM of the denominators
2. **Add or subtract the numerators**
3. The result is over the same denominator (i.e. size of the parts we have added / subtracted)

Example of addition: $\frac{1}{3} + \frac{1}{4}$



$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

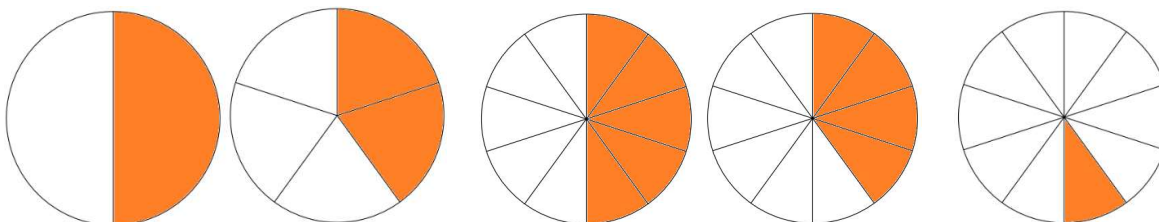
1. Express all fractions on the same denominator, in this case 12

$$\frac{1}{3} = \frac{4}{12} \text{ and } \frac{1}{4} = \frac{3}{12}$$

2. Add the numerators $4 + 3 = 7$

3. The result is on the same denominator: $\frac{7}{12}$

Example of subtraction: $\frac{1}{2} - \frac{2}{5}$



$$\frac{1}{2} - \frac{2}{5} = \frac{5}{10} - \frac{4}{10} = \frac{1}{10}$$

1. Express all fractions on the same denominator, in this case 10

$$\frac{1}{2} = \frac{5}{10} \text{ and } \frac{2}{5} = \frac{4}{10}$$

2. Subtract the numerators $5 - 4 = 1$

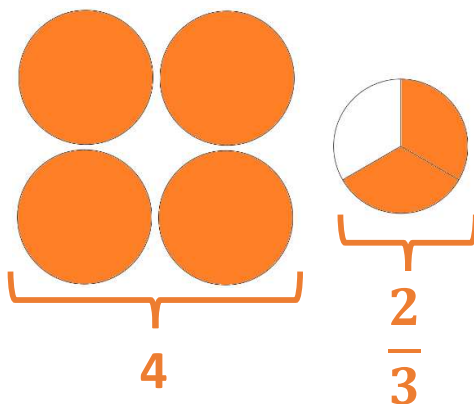
3. The result is on the same denominator: $\frac{1}{10}$

4.1.4 Mixed numbers and improper fractions



A **mixed number** is a number made of a whole number and a fraction – it is the sum of these.

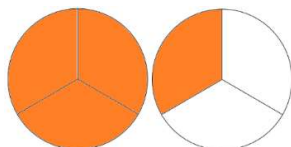
Example: $4\frac{2}{3} = 4 + \frac{2}{3}$



An **improper fraction** is a fraction where the numerator (top) is bigger than the denominator (bottom). It is sometimes called a **top-heavy fraction**.

Thinking of what a fraction is, when the top is bigger than the bottom, it means that we are considering more parts (top) than there are in total in one whole (bottom). So, we have **more than one whole**.

Example: $\frac{4}{3}$

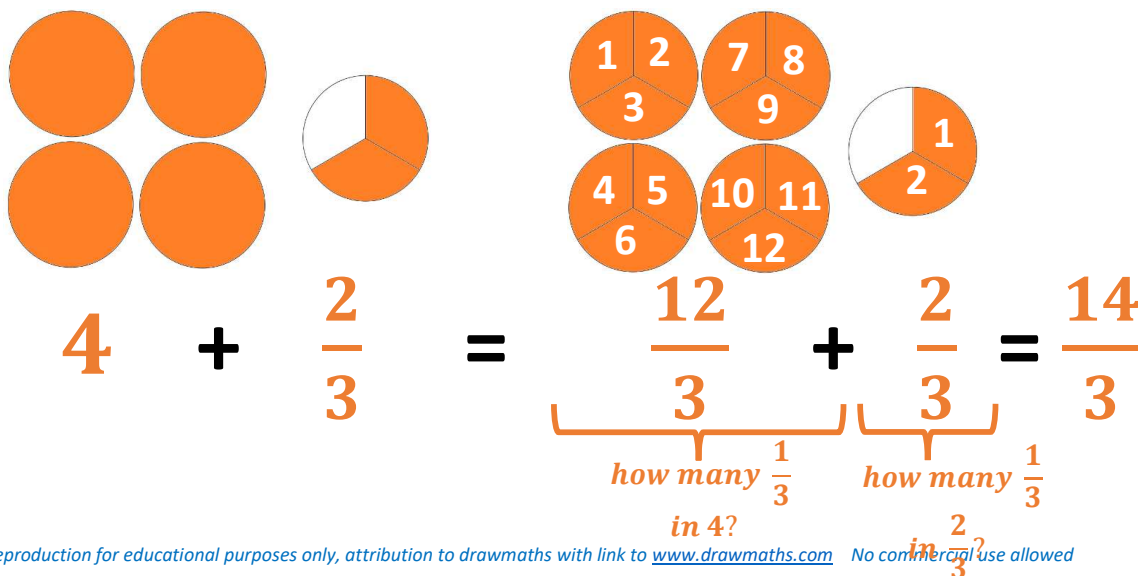


CONVERTING FROM A MIXED NUMBER TO AN IMPROPER FRACTION

1. Write the mixed number as the sum of its whole number part and its fraction part
2. Convert the whole number to a fraction over the same denominator as the fraction part (i.e. work out how many of the fraction-size parts fit in the whole number part?)
3. Add whole part and fraction part, both expressed as fractions

Example: $4\frac{2}{3}$

1. $4\frac{2}{3} = 4 + \frac{2}{3}$
2. $4 = \frac{4}{1} = \frac{12}{3}$
3. $4 + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

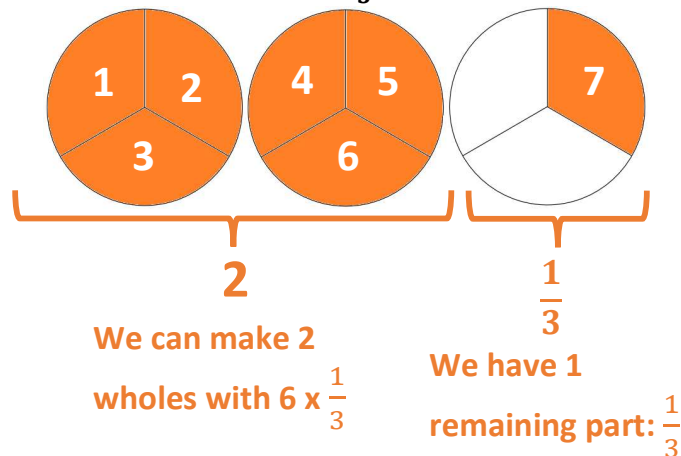


CONVERTING FROM AN IMPROPER FRACTION TO A MIXED NUMBER

1. The denominator (bottom) gives you the number of equal size parts to make one whole
2. Look at the numerator (top), work out **how many wholes** you can make with these parts
3. Express the remaining parts of the numerator over the denominator → **fraction part**

Example: $\frac{7}{3} = 2\frac{1}{3}$

1. I need **3 parts to make 1 whole**, as 3 thirds is 1, $3 * \frac{1}{3} = 1$
2. There are 7 parts at the top, I can use 6 parts to make 2 wholes as $6 = 2 * 3$ → the **whole part is 2**
3. I have 1 part remaining → the **fraction part is $\frac{1}{3}$**

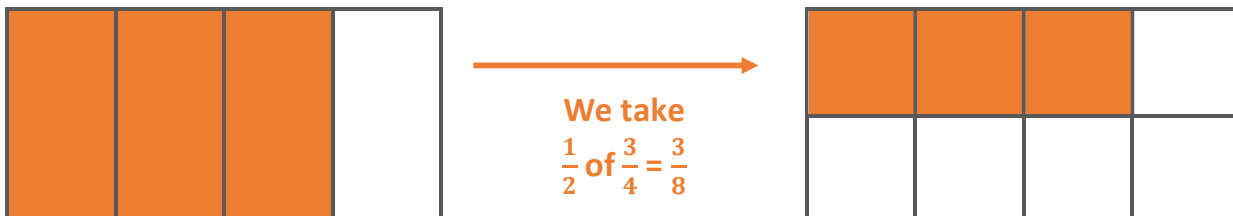


4.1.5 Multiplying fractions

Multiplying fractions is like **taking a fraction of a fraction**.

Example: $\frac{1}{2} * \frac{3}{4} = \frac{3}{8}$

- Each quarter is halved so we have eighths. The denominators are multiplied together: $2*4=8$
- The numerators are also multiplied together: $1*3=3$



MULTIPLYING FRACTIONS

1. Multiply the numerators together and the denominators together i.e. multiply top and bottom
2. Simplify the resulting fraction if possible

Example: $\frac{3}{5} * \frac{2}{9} = \frac{3*2}{5*9} = \frac{6}{45} = \frac{2}{15}$

TIP1: to multiply a fraction by a whole number, divide the whole number by 1 to convert it into a fraction.

Example: $\frac{3}{8} * 2 = \frac{3}{8} * \frac{2}{1} = \frac{3*2}{8*1} = \frac{6}{8} = \frac{3}{4}$

TIP2: to multiply by a mixed number, convert it to an improper fraction, and multiply top and bottom.

Example: $\frac{3}{8} * 2\frac{1}{3} = \frac{3}{8} * \frac{7}{3} = \frac{21}{24} = \frac{7}{8}$

4.1.6 Dividing fractions

Dividing by a number is making groups of the size of this number. This works for fractions too.

Example: $3\frac{1}{2} \div \frac{1}{4}$ or how many groups of $\frac{1}{4}$ can I make with $3\frac{1}{2}$. The answer is 14.



But it can be difficult to visualise the answer as above, so we use the **Keep, Change, Flip method (KCF)**

DIVIDING FRACTIONS

1. Rewrite the division sentence underneath, doing the following
 - **KEEP** the first fraction as it is
 - **CHANGE** the division sign for a multiplication sign: $\div \rightarrow \times$
 - **FLIP** the numerator and denominators of the second fraction
2. Then carry out the multiplication as usual, multiplying top and bottom.

Example: $\frac{13}{15} \div \frac{2}{9} =$
 $\frac{13}{15} * \frac{9}{2} = \frac{13*9}{15*2} = \frac{117}{30} = \frac{39}{10}$

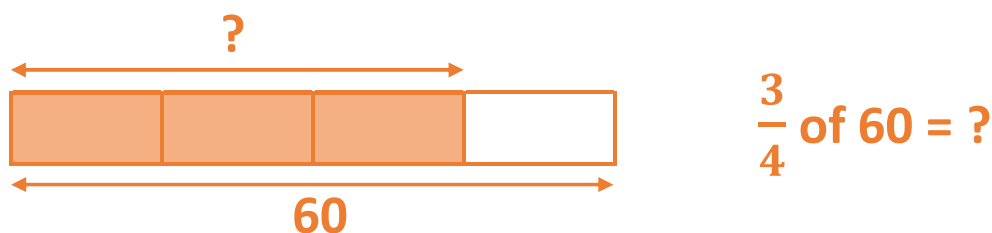
4.1.7 Fractions of amounts

FRACTIONS OF AMOUNTS

1. Draw a fraction bar and write the information you know
 - **FRACTION:** split the fraction bar and shade in the fraction mentioned in the text
 - **SHADED:** mark the length of the shaded area with the amount or '?' if unknown
 - **TOTAL:** mark the length of the fraction bar with the total amount or '?' if unknown.
2. Work out the value of **one part** by using the information given
3. Multiply the value of one part by the **number of parts needed** to answer the question

Example 1: $\frac{3}{4}$ of 60 = ?

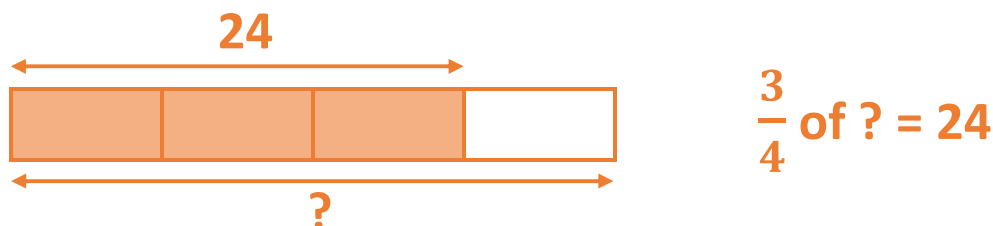
1. I draw the fraction bar and enter the information I know:
 - a. I split the fraction bar in 4 and shade in 3 parts out of 4
 - b. I mark the length of 3 parts with a '?' as it is unknown
 - c. I mark the length of the fraction bar with 60 as it is the total amount



2. I work out the value of one part. I know that 4 parts = 60, so 1 part = $\frac{60}{4} = 15$
3. I multiply the value of one part by 3, as I am looking for $\frac{3}{4}$ so $3 * 15 = 45$
So $\frac{3}{4}$ of 60 = 45

Example 2: $\frac{3}{4}$ of ? = 24

1. I draw the fraction bar and enter the information I know:
 - a. I split the fraction bar in 4 and shade in 3 parts out of 4
 - b. I mark the length of 3 parts with 24 as it is the value given as the answer
 - c. I mark the length of the fraction bar with '?' as it is unknown



2. I work out the value of one part. I know that 3 parts = 24, so 1 part = $\frac{24}{3} = 8$
3. I multiply the value of one part by 4, as I am looking for the total i.e. 4 parts so $4 * 8 = 32$
So $\frac{3}{4}$ of 32 = 24

4.2 Decimals

4.2.1 What is a decimal?



A decimal is a number made of two parts:

- a **whole number part**, before the decimal point
- a **fractional part**, after the decimal point

Example: 12.83 is a decimal:

- 12 is its whole number part
- .83 is its fractional part

We use decimals when **part of a number is less than one**.

Digits in the fractional part have a place value of $\frac{1}{\text{power of } 10}$, as shown in the place value frame below:

T	O	•	t	h
TENS	ONES		tenths	hundredths
10	1		$\frac{1}{10} = 0.1$	$\frac{1}{10^2} = \frac{1}{100} = 0.01$
1	2	•	8	3

As mentioned earlier, you can **use the place value frame to represent decimals**. (here 1=1 big square)

Example: 11.482 is made of

- 1 ten (T) = $1 \times 10 = 10$
- 1 one (O) = $1 \times 1 = 1$
- 4 tenths (th) = $4 \times 0.1 = 0.4$
- 8 hundredths (h) = $8 \times 0.01 = 0.08$
- 2 thousandths (th) = $2 \times 0.001 = 0.002$

T	O	•	t	h	th
TENS	ONES		tenths	hundredths	thousandths
10	1		$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1,000} = 0.001$
1	1	•	4	8	2

4.2.2 Recurring decimal



A recurring decimal is a decimal whose fractional part has **one or more digits that repeat indefinitely**. We place a dot over the first and last digits of the recurring group of digits.

Examples: $0.888888 \dots = 0.\dot{8}$ $2.2454545 \dots = 2.2\dot{4}\dot{5}$ $0.0123123123 \dots = 0.0\dot{1}\dot{2}\dot{3}$

It is possible to remove the recurring part by converting a recurring decimal into a fraction.

CONVERTING A RECURRING DECIMAL INTO A FRACTION

1. Name your recurring decimal x and write $x =$ recurring decimal
2. Underneath, and lining up the place values, write the value of $10x$, $100x$, etc. **until the fractional parts of two of the expressions of x are equal**. Write the same number of place values in the fractional part, noting the recurring part on the digits most to the right.
3. Subtract the two expressions of x with identical fractional parts, on both sides of the equations
4. Work out the value of x as a fraction and simplify if possible.

Examples:

$$\begin{array}{rcl} x & = & 0.\dot{8} \\ 10x & = & 8.\dot{8} \\ 10x - x & = & 8 \\ 9x & = & 8 \\ x & = & \frac{8}{9} \end{array}$$

$$\begin{array}{rcl} x & = & 2.\dot{2}\dot{4}\dot{5} \\ 10x & = & 22.\dot{4}\dot{5}\dot{4} \\ 100x & = & 224.\dot{5}\dot{4}\dot{5} \\ 1000x & = & 2,245\dot{4}\dot{5}\dot{4} \\ 1,000x - 10x & = & 2,223 \\ 990x & = & 2,223 \\ x & = & \frac{2,223}{990} \\ x & = & \frac{247}{110} \end{array}$$

$$\begin{array}{rcl} x & = & 0.\dot{0}\dot{1}\dot{2}\dot{3} \\ 10x & = & 0.\dot{1}\dot{2}\dot{3}\dot{1} \\ 100x & = & 1.\dot{2}\dot{3}\dot{1}\dot{2} \\ 1000x & = & 12.\dot{3}\dot{1}\dot{2}\dot{3} \\ 10,000x & = & 123.\dot{1}\dot{2}\dot{3}\dot{1} \\ 10,000x - 10x & = & 123 \\ 9,990x & = & 123 \\ x & = & \frac{123}{9,990} \\ x & = & \frac{41}{3,330} \end{array}$$

4.3 Percentages



A percentage is a fraction **out of 100** (i.e. with 100 at the denominator).

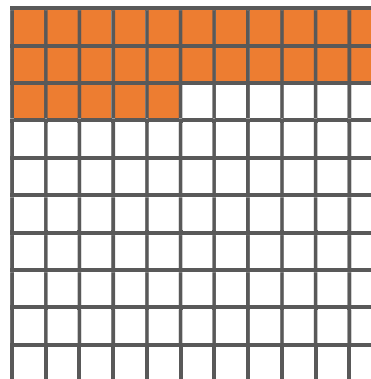
In a percentage, the focus is on the term **100**. The word 'cent' in French means 'hundred'.

Percent means **per 100**. So you can always **replace '%' by $\frac{\quad}{100}$**

Example: 25%

$$25\% = \frac{25}{100} \text{ — this is 25 out of 100}$$

We represent it as 25 squares out of a 100.



4.4 Converting between fractions, decimals, and percentages

Fractions, decimals and percentages are **expressing the same value in different ways**. We can convert from one to another using the frames on the next pages: 1st is the method, 2nd an example with visual.

But **let's first look at how common values** express as a fraction, as a decimal, and as a percentage:

Fraction /10	Visual /10	Decimal	Visual /100	Fraction /100	Percentage
$\frac{0}{10}$		0		$\frac{0}{100}$	0%
$\frac{1}{10}$		0.1		$\frac{10}{100}$	10%
$\frac{2}{10}$		0.2		$\frac{20}{100}$	20%
$\frac{3}{10}$		0.3		$\frac{30}{100}$	30%
$\frac{4}{10}$		0.4		$\frac{40}{100}$	40%
$\frac{5}{10}$		0.5		$\frac{50}{100}$	50%
$\frac{6}{10}$		0.6		$\frac{60}{100}$	60%
$\frac{7}{10}$		0.7		$\frac{70}{100}$	70%
$\frac{8}{10}$		0.8		$\frac{80}{100}$	80%
$\frac{9}{10}$		0.9		$\frac{90}{100}$	90%
$\frac{10}{10}$		1.0		$\frac{100}{100}$	100%

When we convert from a fraction, we often try to express it over 100, or over 1000. It is useful to know the **factors of 100 and 1000**.

N.B. numbers with a prime factor of 3 or 7 do not go into 100 or 1,000.

FACTORS OF 100

$$100 = 2 \times 50$$

$$100 = 4 \times 25$$

$$100 = 5 \times 20$$

$$100 = 10 \times 10$$

FACTORS OF 1,000

$$1,000 = 2 \times 500$$

$$1,000 = 4 \times 250$$

$$1,000 = 5 \times 200$$

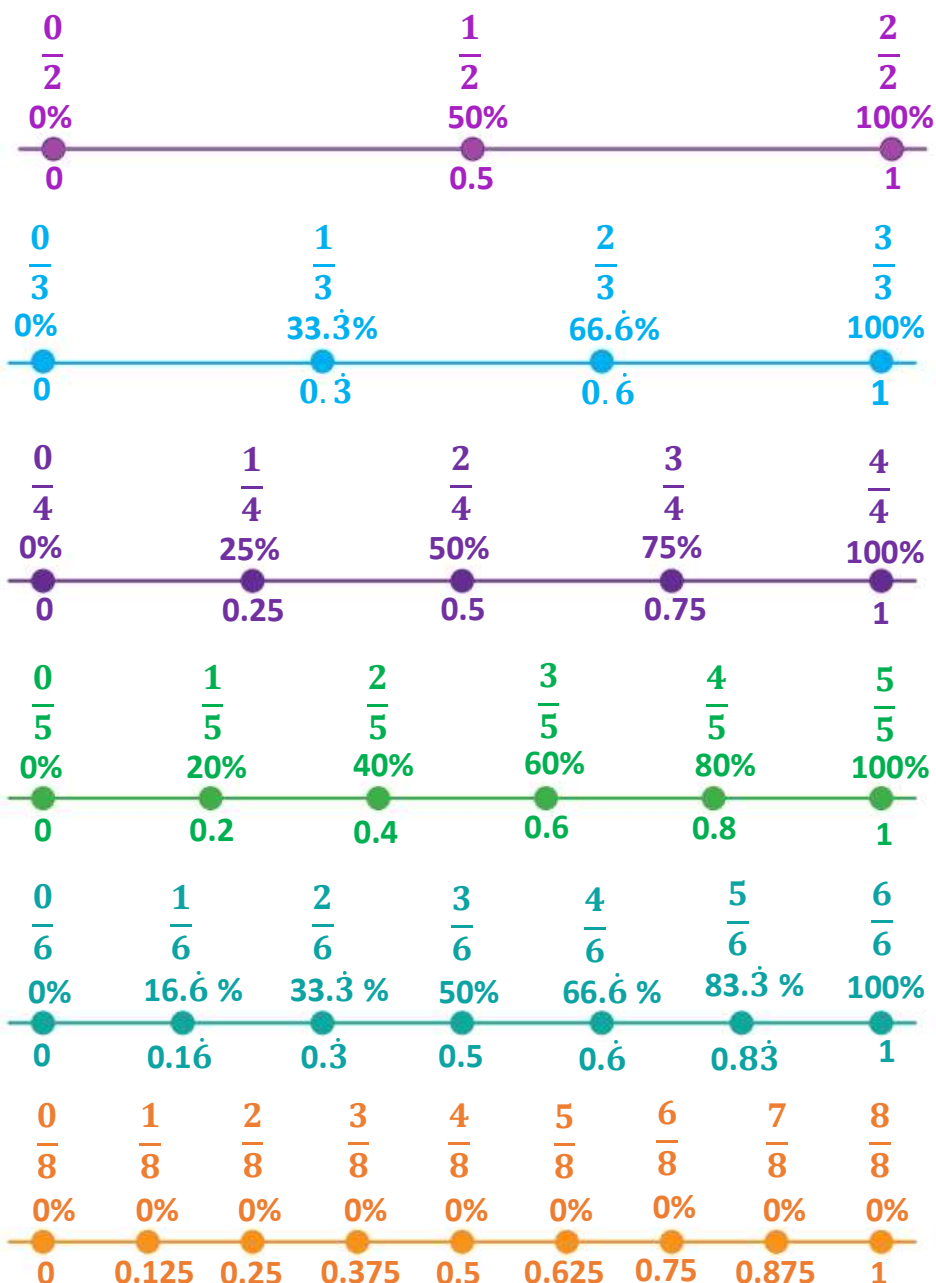
$$1,000 = 8 \times 125$$

$$1,000 = 10 \times 100$$

$$1,000 = 20 \times 50$$

$$1,000 = 25 \times 40$$

Below are corresponding **percentages, decimals and fractions on number lines**:



<p style="text-align: center;">Fraction → Decimal</p> $\frac{7}{20} = 0.35$	<p style="text-align: center;">Decimal → Percentage</p> $0.253 = 25.3\%$	<p style="text-align: center;">Percentage → Fraction</p> $25\% = \frac{1}{4}$
<ol style="list-style-type: none"> Find an equivalent fraction with the denominator = 10, 100, 1,000 etc. $\frac{7}{20} = \frac{35}{100}$ Transfer in the right place value position $\frac{35}{100}$ is 35 hundredths Read the decimal as in the frame 0.35 <p>Otherwise: Divide numerator by denominator adding 0s after the decimal point if needed</p>	<ol style="list-style-type: none"> Write decimal in the place value frame Express in hundredths 0.253 as expressed in hundredths $0.253 = 25.3 \times \frac{1}{100} = \frac{25.3}{100}$ A number over 100 is a percentage (/100 = per 100 = 'percent' = %) $\frac{25.3}{100} = 25.3\%$ 	<ol style="list-style-type: none"> Write the percentage as over 100 (% = 'percent' = per 100 = /100) $25\% = \frac{25}{100}$ Simplify the fraction (if possible) $\frac{25}{100} = \frac{1}{4}$ (divide top & bottom by 25)
<p style="text-align: center;">Fraction → Percentage</p> $\frac{1}{8} = 12.5\%$	<p style="text-align: center;">Decimal → Fraction</p> $0.364 = \frac{91}{250}$	<p style="text-align: center;">Percentage → Decimal</p> $12.6\% = 0.126$
<ol style="list-style-type: none"> Find an equivalent fraction with the denominator = 10, 100, 1,000 etc. $\frac{1}{8} = \frac{125}{1,000}$ Transfer in the right place value position $\frac{125}{1,000}$ is 125 thousandths Read the decimal as hundredths, add % (/100 = per 100 = 'percent' = %) 12.5% <p>Otherwise: Divide numerator by denominator adding 0s after the decimal point if needed & read the decimal as hundredths & add %</p>	<ol style="list-style-type: none"> Express the decimal as a whole number in the place value of its last digit 0.364 → 364 thousandths: $0.364 = 364 \times \frac{1}{1,000} = \frac{364}{1,000}$ Simplify the fraction (if possible) $\frac{364}{1,000} = \frac{91}{250}$ (divide top & bottom by 4) 	<ol style="list-style-type: none"> Write the percentage as over 100 (% = 'percent' = per 100 = /100) $12.6\% = \frac{12.6}{100}$ As it's over 100, write numerator as if a whole number in the 'hundredth' column Read the decimal as in the frame 0.126

Fraction → Decimal $\frac{7}{20} = 0.35$	Decimal → Percentage $0.253 = 25.3\%$	Percentage → Fraction $25\% = \frac{1}{4}$																																																
<p>1. Find an equivalent fraction on 100 $\frac{7}{20} = \frac{35}{100}$</p> <p>2. $\frac{35}{100}$ is 35 hundredths</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 25%;">T</td> <td style="width: 25%;">O</td> <td style="width: 25%;">t</td> <td style="width: 25%;">h</td> </tr> <tr> <td><small>TENS</small></td> <td><small>ONES</small></td> <td><small>tenths</small></td> <td><small>hundredths</small></td> </tr> <tr> <td>10</td> <td>1</td> <td>$\frac{1}{10} = 0.1$</td> <td>$\frac{1}{100} = 0.01$</td> </tr> <tr style="font-size: 2em;"> <td>0</td> <td>.</td> <td>3</td> <td>5</td> </tr> </table> <p>3. Read the decimal as in frame 0.35</p>	T	O	t	h	<small>TENS</small>	<small>ONES</small>	<small>tenths</small>	<small>hundredths</small>	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	0	.	3	5	<p>1. Write decimal in the frame</p> <p>2. Express in hundredths $0.253 = 25.3 \times \frac{1}{100} = \frac{25.3}{100}$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 25%;">O</td> <td style="width: 25%;">t</td> <td style="width: 25%;">h</td> <td style="width: 25%;">th</td> </tr> <tr> <td><small>ONES</small></td> <td><small>tenths</small></td> <td><small>hundredths</small></td> <td><small>thousandths</small></td> </tr> <tr> <td>1</td> <td>$\frac{1}{10} = 0.1$</td> <td>$\frac{1}{100} = 0.01$</td> <td>$\frac{1}{1,000} = 0.001$</td> </tr> <tr style="font-size: 2em;"> <td>0</td> <td>.</td> <td>2</td> <td>5</td> </tr> </table> <p>3. A number over 100 is a percentage $\frac{25.3}{100} = 25.3\%$</p>	O	t	h	th	<small>ONES</small>	<small>tenths</small>	<small>hundredths</small>	<small>thousandths</small>	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1,000} = 0.001$	0	.	2	5	<p>1. Write the percentage as over 100 (% = 'percent' = per 100 = /100) $25\% = \frac{25}{100}$</p> <p>2. If numerator is a decimal, multiply it by 10, 100, 1000 to make it a whole number. Multiply the denominator by the same number so it is an equivalent fraction. $12.5\% = \frac{12.5}{100} = \frac{125}{1000}$ (x top & bottom by 10)</p> <p>3. Simplify the fraction (if possible) $\frac{25}{100} = \frac{1}{4}$ (÷ top & bottom by 25)</p>																
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5 Powers and roots

5.1 Powers

5.1.1 What is a power?



The terms **power**, **exponent**, **index** are often used interchangeably, but in mathematics we use:

- **power** to define the concept of a base number being raised to an exponent / index
- **exponent** or **index** to indicate the number of times the base is multiplied by itself

The **exponent / index** of a number (**base**) shows **how many times the base is multiplied by itself**. It is written in small on the top right of the base.

base **exponent** 5 multiplied by itself 2 times

We say: “5 to the power of 2” or “5 raised to the power of 2” or “5 squared”. It means: “The base number 5 has been multiplied by itself 2 times”

Examples:

- $6^3 = 6 * 6 * 6 = 216$
- $10^4 = 10 * 10 * 10 * 10 = 10,000$
- $(-1)^2 = (-1) * (-1) = +1$
- $x^5 = x * x * x * x * x$

Why and when do we use powers? It is **simpler to write/read** 10^5 than $10 * 10 * 10 * 10 * 10$ or 100,000
Scientists write massive and miniscule numbers using the **standard form**

5.1.2 Index rules

N.B. You do not need to learn all the index rules below by heart, some rules can be deduced by creating a simple example and solving it using your knowledge of numbers.

Zero rule $a^0 = 1$	Identity rule $a^1 = a$
Any number to the power of 0 equals 1. $18^0 = 1$ <u>Proof:</u> $18^2 * 18^0 = 18^{2+0} = 18^2$ so $18^0 = 1$ (please see product rule below)	Any number to the power of 1 equals the number $7^1 = 7$ <u>Proof:</u> $7^1 = 7$ (7 multiply itself once)

<p style="text-align: center;">Product rule</p> $a^m * a^n = a^{m+n}$	<p style="text-align: center;">Quotient rule</p> $\frac{a^m}{a^n} = a^{m-n}$
<p>The product of two numbers with same base a but raised to different powers m and n equals the base a raised to the sum of the powers m+n</p> $3^2 * 3^4 = 3^{2+4} = 3^6$ <p><u>Proof:</u> $3^2 * 3^4 = 3 * 3 * 3 * 3 * 3 * 3 = 3^6$</p>	<p>The quotient of two numbers with same base a but raised to different powers m and n equals the base a raised to the difference of the powers m-n</p> $\frac{3^5}{3^2} = 3^{5-2} = 3^3$ <p><u>Proof:</u> $\frac{3^5}{3^2} = \frac{\cancel{3} * \cancel{3} * 3 * 3 * 3}{\cancel{3} * \cancel{3}} = 3 * 3 * 3 = 3^3$</p>
<p style="text-align: center;">Power of a product</p> $(a * b)^m = a^m * b^m$	<p style="text-align: center;">Power of a quotient</p> $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
<p>The product of two numbers a x b raised to the power of m equal the product of each these numbers raised to the power of m. We 'distribute' the power to each number in the product.</p> $(3 * 5)^2 = 3^2 * 5^2 = 9 * 25 = 225$ <p><u>Proof:</u> $(3 * 5)^2 = (3 * 5) * (3 * 5) = 3 * 5 * 3 * 5 = 3 * 3 * 5 * 5 = 3^2 * 5^2$</p>	<p>The quotient of two numbers $\frac{a}{b}$ raised to the power of m equal the quotient of each these numbers raised to the power of m. We 'distribute' the power to each number in the quotient.</p> $\left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2}$ <p><u>Proof:</u> $\left(\frac{4}{3}\right)^2 = \frac{4}{3} * \frac{4}{3} = \frac{4*4}{3*3} = \frac{4^2}{3^2}$</p>
<p style="text-align: center;">Power of a power</p> $(a^m)^n = a^{m*n}$	<p style="text-align: center;">Negative power</p> <div style="text-align: center; border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">H</div> $a^{-m} = \frac{1}{a^m}$
<p>A number a to the power of m raised to the power of n equals a raised to the power of m*n</p> $(4^2)^3 = 4^{2*3} = 4^6$ <p><u>Proof:</u> $(4^2)^3 = (4 * 4)^3 = (4 * 4) * (4 * 4) * (4 * 4) = 4 * 4 * 4 * 4 * 4 * 4 = 4^6$</p>	<p>A number a raised to a negative power -m equals 1 divided by the same number a raised to the same power, but of opposite sign +m</p> $9^{-2} = \frac{1}{9^{+2}} = \frac{1}{9 * 9} = \frac{1}{81}$ <p><u>Proof:</u> $9^{-2} * 9^2 = 9^{-2+2} = 9^0 = 1$</p>
<p style="text-align: center;">Fractional power</p> $a^{\frac{1}{m}} = \sqrt[m]{a}$	<p style="text-align: center;">Two-stage fractional power</p> $a^{\frac{p}{m}} = (\sqrt[m]{a})^p$
<p>If the exponent of a number a is a fraction such as $\frac{1}{m}$, m is the order the root of a</p> $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ <p>27 to the power of $\frac{1}{3}$ = cubic root ($\sqrt[3]{\quad}$) of 27 (i.e. root of order 3 of number 27)</p> <p><u>Proof:</u> these two expressions are equal:</p> <ul style="list-style-type: none"> - $(27^{\frac{1}{3}})^3 = 27^{\frac{1}{3}} * 27^{\frac{1}{3}} * 27^{\frac{1}{3}} = (27^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}) = 27^1 = 27$ - $(\sqrt[3]{27})^3 = 27$ 	<p>If the exponent of a number a is a fraction such as $\frac{p}{m}$, m is the order the root of a and p is the power this root is raised to. Follow 2 stages:</p> <ol style="list-style-type: none"> look for the m-degree root of number a, raise the answer to the power p $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$ <ol style="list-style-type: none"> The cubic (degree 3) root of 64 is 4 ($\sqrt[3]{64} = 4$) Raise the answer to the power of 2 ($4^2 = 16$) <p><u>Proof:</u> $64^{\frac{2}{3}} = 64^{\frac{1}{3} * 2} = (64^{\frac{1}{3}})^2 = (\sqrt[3]{64})^2$</p>



5.2 Roots and surds

5.2.1 What is a root?

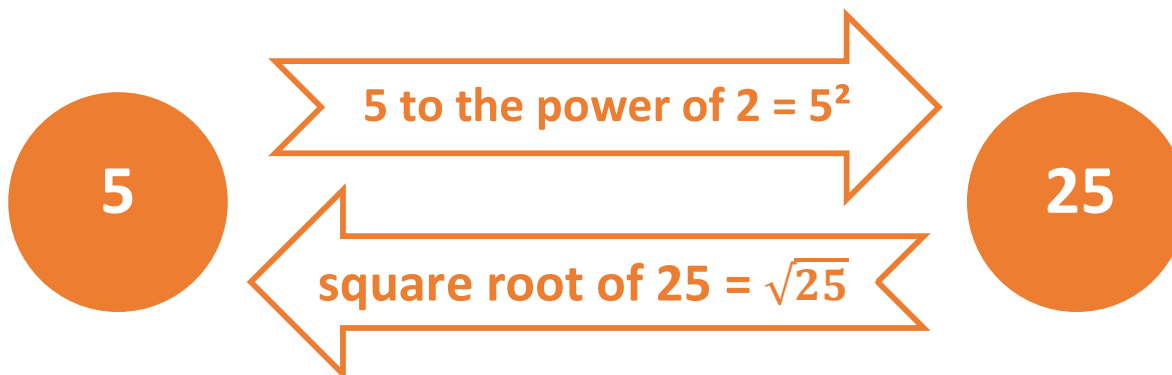


Roots are the reverse process of powers. Roots and powers are **inverse operations**.

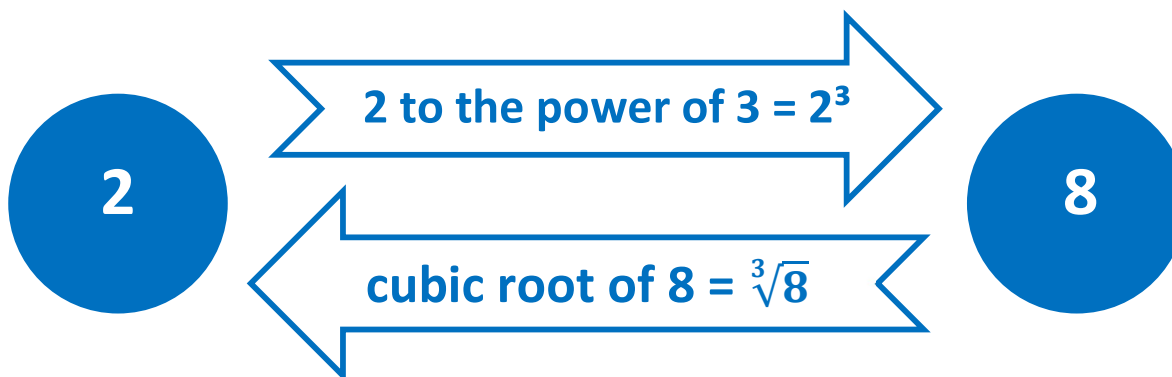
Examples:

If $5^2 = 25$ then $5 = \sqrt[2]{25} = \sqrt{25} \rightarrow$ 5 is the square root of 25, as 5 multiplied by itself 2 times equals 25.

N.B. When a root degree is 2, we omit it. If a root has no degree, it means it is a square root (of degree 2).



If $2^3 = 8$ then $2 = \sqrt[3]{8} \rightarrow$ 2 is the cubic root of 8, as 2 multiplied by itself 3 times equals 8.



We generalise this to any root of degree m :

$$b^m = a$$

$$\sqrt[m]{a} = \sqrt[m]{b^m} = b$$

the m -degree root cancels out with the power of m under the root

The m degree root of a number a is the number b , which multiplied by itself m times equals a

$$\sqrt[m]{a} = \sqrt[m]{b^m} = b$$



5.2.2 What is a surd and how to manipulate surds



A surd is a square root whose answer is not a rational number.

- $\sqrt{9} = 3$, i. e. $\sqrt{9}$ has root of 3, which is a rational number so $\sqrt{9}$ is not a surd.
- $\sqrt{2}$ does not have a proper root so $\sqrt{2}$ is a surd.

H

Square root of a product $\sqrt{a * b} = \sqrt{a} * \sqrt{b}$

The square root of the product of two numbers $a \times b$ equals the product of the square root of each of these numbers. We 'distribute' the square root to each number in the product.

To simplify a surd, find a square number that is a factor of the number under the root. Rewrite the surd as the product of the square number and another number. Distribute the square root, and the square root of the square number will cancel out. Repeat if there are any more factors that are square numbers.

Example: 9 is a square number and factor of 45, so $\sqrt{45} = \sqrt{9 * 5} = \sqrt{9} * \sqrt{5} = 3 * \sqrt{5} = 3\sqrt{5}$

Square root of a quotient $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

The square root of the quotient of two numbers $\frac{a}{b}$ equals the quotient of the square root of each of these numbers. We 'distribute' the square root to each number in the quotient.

Example: $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

Rationalising the denominator

Rationalising means getting rid of the square root at the denominator. In both cases below, you multiply numerator and denominator by the same number/expression, which is the same as multiplying by 1:

- **If the denominator looks like \sqrt{b}**

Method: multiply by $\frac{\sqrt{b}}{\sqrt{b}}$ -

Proof: $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} * \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b^2}} = \frac{a\sqrt{b}}{b}$

There is no square root left on the denominator as the square root of a square number is the number itself.

Example: $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5^2}} = \frac{2\sqrt{5}}{5}$

- **If the denominator looks like $a \pm \sqrt{b}$**

Method: multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$, (but opposite sign between the 2 terms)

Proof: $\frac{1}{a + \sqrt{b}} = \frac{1}{a + \sqrt{b}} * \frac{a - \sqrt{b}}{a - \sqrt{b}} = \frac{1 * (a - \sqrt{b})}{(a + \sqrt{b}) * (a - \sqrt{b})} = \frac{a - \sqrt{b}}{a^2 - b}$

There is no square root on the denominator. This is because we applied a rule known as the Difference of Two Squares (DOTS), which says that $(a+b)*(a-b)=a^2-b^2$ because $(a+b)*(a-b)=a^2 + a^b - a^b + b^2$

Example: $\frac{6}{3 + \sqrt{2}} = \frac{6}{3 + \sqrt{2}} * \frac{3 - \sqrt{2}}{3 - \sqrt{2}} = \frac{6 * (3 - \sqrt{2})}{(3 + \sqrt{2}) * (3 - \sqrt{2})} = \frac{18 - 6\sqrt{2}}{3^2 - (\sqrt{2})^2} = \frac{18 - 6\sqrt{2}}{9 - 2} = \frac{18 - 6\sqrt{2}}{7}$

6 Basic skills and top tips

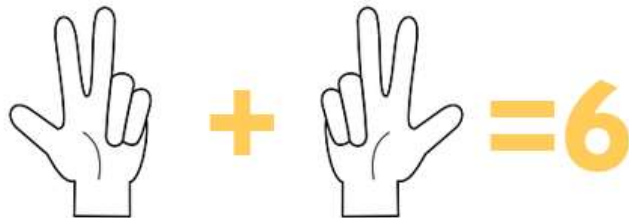
6.1 Basic skills: doubling and halving

6.1.1 Doubling numbers up to 10

To double numbers up to 10, we use mirror hands.

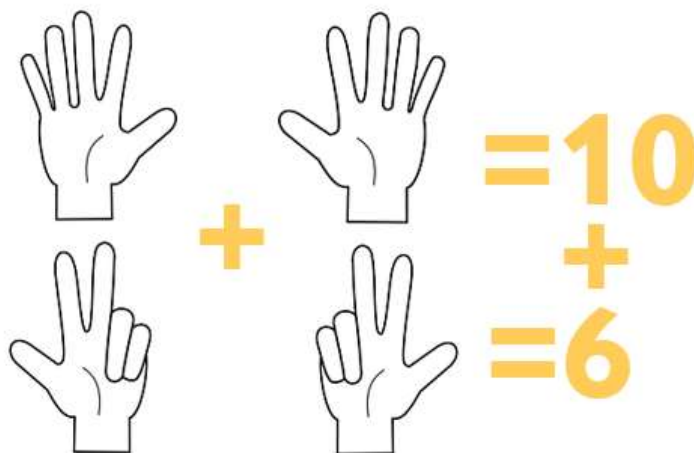
Show the number with your fingers and imagine the reflection of your hands in a mirror i.e. your mirror hands. Between your own fingers and your mirror hands fingers, you have double the number of fingers.

- **For numbers 0-5**, you can use your second hand as the mirror hand and add up your fingers.
For example: double 3 is 6.



- **For numbers 5-10**, you will have one hand with 5 fingers and one hand with the remaining fingers. Doubling the hand with 5 fingers will give you 10. So, it is then easy to just double the other hand and add the result to 10.

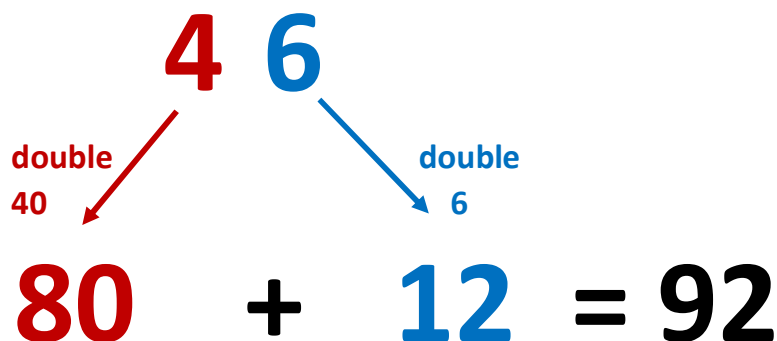
For example: double 8 is 16 (double 5 is 10 and double 3 is 6, add them up $10+6 = 16$)



6.1.2 Doubling numbers above 10

To double numbers above 10, double each digit, in its own place value, and add up the results.

Example: double 46 is 92.



6.1.3 Halving numbers

To halve numbers, halve each digit, in its own place value, and add up the results.

For example: half of 74 is 37.

$$\begin{array}{c}
 74 \\
 \swarrow \text{halve } 70 \quad \searrow \text{halve } 4 \\
 35 + 2 = 37
 \end{array}$$

TIP: to halve 70, you might want to break it down in $60 + 10$, halve each part and add the halves up:

$$\begin{array}{c}
 60 + 10 = 70 \\
 \swarrow \text{halve } 60 \quad \searrow \text{halve } 10 \quad \searrow \text{halve} \\
 30 + 5 = 35
 \end{array}$$

6.2 Basic skills: times tables

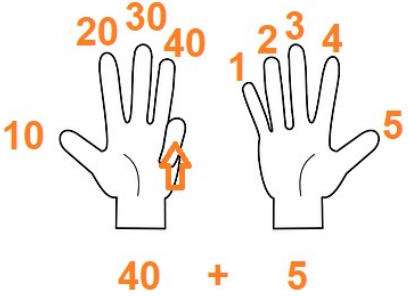
The objective is to **know your times tables by heart**, so you can recall the answer, without counting on. There are some great apps for you to practise your times tables e.g. TT Rock star, Squeebles, etc. Spending time learning times tables is beneficial. It saves you time in the exam and prevent mistakes. If you are struggling to learn your times tables, below are some tricks that might help.

Tip 1: A good order to learn times tables is 0x, 1x, 2x, 10x, 5x, 3x, 4x, 9x, 11x, 12x, 6x, 7x, 8x.

Tip 2: Remember, $4 \times 5 = 5 \times 4$ – swap the order, you get the same result. Multiplication is *commutative*.

Tip 3: there are songs to learn times tables e.g. <https://www.youtube.com/watch?v=uVOZL2h8IRg>

	Trick	Visual
0X	= 0 0 lot of something is 0 (zero lots of something = zero) Any lots of 0 is 0 (any lots of nothing is nothing)	$2 \times \bigcirc = \bigcirc \bigcirc = \bigcirc$
1X	= number 1 lot of 2 is 2 1 lot of 3 is 3 Etc.	$1 \times \bigcirc \times \bigcirc = \bigcirc \times \bigcirc$ $1 \times \bigcirc \times \bigcirc \times \bigcirc = \bigcirc \times \bigcirc \times \bigcirc$
2X	= double the number (mirror hands) To double, show the number with your fingers and imagine mirror hands. For numbers greater than 5, remember that $2 \times 5 = 10$ (a full hand and its mirror hand make 10), so double the small number (above 5) and add to 10	$\text{Hand} + \text{Hand} = 6$ $2 \times 3 = 6$ $\text{Hand} + \text{Hand} = 10$ $\text{Hand} + \text{Hand} = 6$ $2 \times 8 = 16$
3X	= double and add 1 more 3×7 - Double 7 is 14 - Add 1 extra 7 → $14 + 7 = 21$	$3 \times 7 =$ $2 \times 7 = 14$ $+ 1 \times 7 = 7$ $3 \times 7 = 14 + 7 = 21$
4X	= double the number + double again 4×8 - Double 8 is 16 - Double 16 is 32	$2 \times 8 = 16$ $\text{Hand} + \text{Hand} = 10$ $\text{Hand} + \text{Hand} = 6$ $2 \times 8 = 16$ $2 \times 16 = 32$ $\begin{array}{c} 16 \\ \swarrow \text{double } 10 \\ 20 \\ \searrow \text{double } 6 \\ 12 \end{array}$ $20 + 12 = 32$
5X	= Pair 5s → 10. Add 5 if a 5 left. Imagine each finger equals 5. - Put as many fingers up as there are 5s - Pair them up to make 10s. - Add 5 if a finger is not paired up.	 $5 \times 7 = 35$

6X	<p>= learn these facts</p> <ul style="list-style-type: none"> - $6 \times 6 = 36$ - $6 \times 7 = 42$ and $7 \times 6 = 42$ - $6 \times 8 = 48$ and $8 \times 6 = 48$ <p>(Notice 6×6 ends in 6, and 6×8 ends in 8)</p> <p>OR = X5 + 1 more</p> <p>Multiply by 5 and add one more</p> $6 \times 8 = ?$ <ul style="list-style-type: none"> - $5 \times 8 = 40$ - $1 \times 8 = 8$ - $6 \times 8 = 40 + 8 = 48$ 	$6 \times 6 = 36$ $6 \times 7 = 7 \times 6 = 42$ $6 \times 8 = 8 \times 6 = 48$ $6 \times 8 = ?$ <ul style="list-style-type: none"> - $5 \times 8 = 40$ - $1 \times 8 = 8$ - $6 \times 8 = 40 + 8 = 48$
7X	<p>= learn these facts</p> <ul style="list-style-type: none"> - $7 \times 7 = 49$ - $7 \times 8 = 56$ (it goes 5-6-7-8 answer to question) 	$7 \times 7 = 49$ $7 \times 8 = 56$
8X	<p>= learn this fact</p> <ul style="list-style-type: none"> - $8 \times 8 = 64$ 	$8 \times 8 = 64$
9X	<p>= fingers up except the number of 9, fingers before = 10, fingers after = 1</p> <p>9×5</p> <ul style="list-style-type: none"> - 5th finger down, starting from the left - 4 fingers before it, count in 10s = 10, 20, 30, 40 - 5 fingers after it, count in 1s = 1, 2, 3, 4, 5 - Add them up: $9 \times 5 = 40 + 5 = 45$ 	
10X	<p>= add 0 after the number</p> $10 \times 8 = 80$	$10 \times 2 = 20$ $10 \times 3 = 30$ $10 \times 4 = 40$ Etc.
11X	<p>= up to 9, repeat number twice</p> $11 \times 9 = 99$ <p>= 10+, create a space between the 2 digits, add sum of digits in between</p> $11 \times 12 = 132$ ($3 = 1+2$) N.B. carry over 10 if sum of digits is above 10	$11 \times 7 = 77$ $11 \times 15 = 165$ ($6 = 1+5$)
12X	<p>= $10X + 2X$</p> <p>$10 \times$ = Multiply number by 10 (add zero at the end)</p> <p>$2 \times$ = Double the number (mirror hands)</p> <p>$10 \times + 2 \times$: Add these together</p> $12 \times 6 = 10 \times 6 + 2 \times 6 = 60 + 12 = 72$	$12 \times 12 = 144$ <ul style="list-style-type: none"> - $10 \times 12 = 120$ - $2 \times 12 = 24$ - $120 + 24 = 144$

6.3 Basic skills: adding and subtracting within 20

6.3.1 Bridging 10

ADDING OR SUBTRACTING WITHIN 20 BY BRIDGING 10

Use bridging 10 to add or subtract numbers if the process makes you cross 10.

1. Start from the first number and add/subtract only part of the second number, so you reach 10*
2. Add/subtract whatever is left of the second number, starting from 10
3. You land on the answer

(* Use your number bonds to work out how much to add)

Example: $6 + 7 = ?$

1. Start from 6 and add 4 to reach 10 → $6+4=10$
2. We added 4 instead of 7, so we still need to add 3 more, starting from 10 → $10 + 3 = 13$
3. We land on **13**



Example: $17 - 9 = ?$

1. Start from 17 and subtract 7 to reach 10 → $17 - 7 = 10$
2. We subtracted 7 instead of 9, so we still need to subtract 2 more, starting from 10 → $10 - 2 = 8$
3. We land on **8**

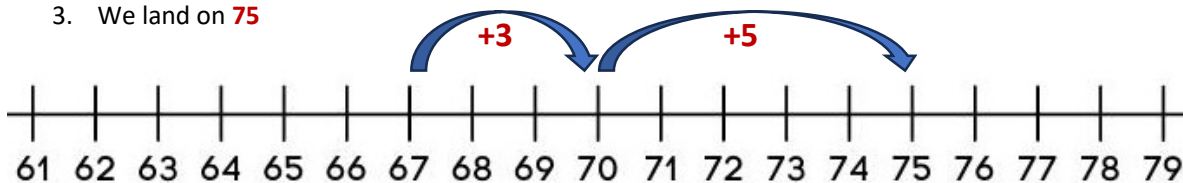


6.3.2 Bridging a ten number

You can **bridge any 10 number**, to help break down your additions and subtractions.

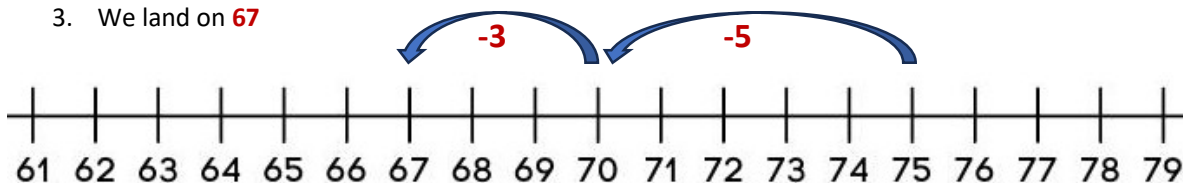
Example: $67+8 = ?$

1. Start from 67 and add 3 to reach the next 10 number, which is 70 → $67+3=70$
2. We added 3 instead of 8, so we still need to add 5 more, starting from 70 → $70 + 5 = 75$
3. We land on **75**



Example: $75-8 = ?$

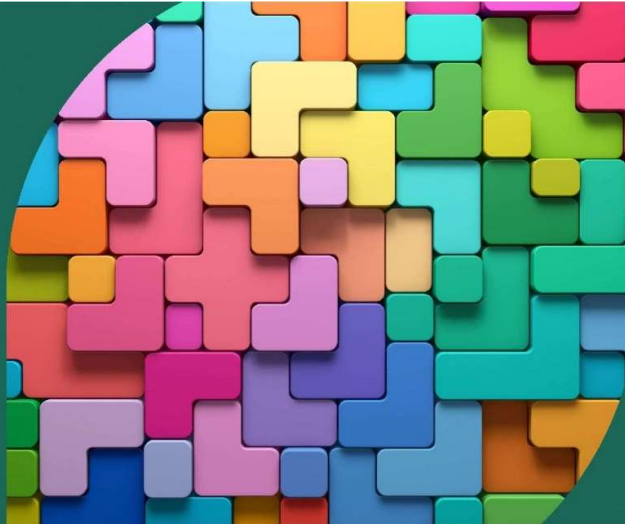
1. Start from 75 and take away 5 to reach the previous 10 number, which is 70 → $75-5=70$
2. We subtracted 5 instead of 8, so we still need to subtract 3 more, starting from 70 → $70 - 3 = 67$
3. We land on **67**



6.3.3 Mental addition within 20

Mental addition within 20

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+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

+ 0 = same

+0 gives the **same** number
 +0 is just adding **nothing**



$$5 + 0 = 5$$

same

$$9 + 0 = 9$$

same

+ 1 = 1 more

+1 gives the **next** number
 +1 gives the **following** number

$$5 + 1 = 6$$

next

$$9 + 1 = 10$$

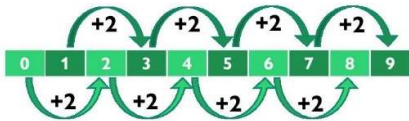
next

+ 2 = next odd or even

+2 gives the **next** odd or even

odd = 1, 3, 5, 7, 9

even = 0, 2, 4, 6, 8



$$3 + 2 = 5$$

next
odd

$$4 + 2 = 6$$

next
even

Bonds to 10 = fill 2 x 5



$$0 + 10 = 10$$



$$1 + 9 = 10$$



$$2 + 8 = 10$$



$$3 + 7 = 10$$



$$4 + 6 = 10$$

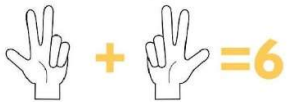


$$5 + 5 = 10$$

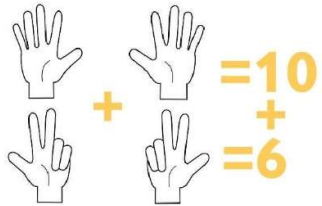
Doubles = mirror hands

To double, show fingers on one hand and add a **mirror hand**

For $5 + 2$ $2 \times 5 = 10$, so double the small number and add to 10



$$3 + 3 = 6$$



$$8 + 8 = 16$$

Near doubles = doubles + 1

To add **2 consecutive numbers**

aka near doubles:

- **double** the lowest number

- **+1** to the result

$$8 + 9 = ?$$



$$8 + 8 = 16$$

$$16 + 1 = 17$$

+ 10 = add 1 ten+10 adds **1 in the Tens** column

$$2 + 10 = 12$$


$$5 + 10 = 15$$


+ 9 = + 10 - 1

+9 is the same as +10 -1

- First +10
- Then -1

$$6 + 9 = ?$$



$$6 + 10 = 16$$

$$16 - 1 = 15$$

+ 8 = + 10 - 2

+8 is the same as +10 -2

- First +10
- Then -2

$$6 + 8 = ?$$



$$6 + 10 = 16$$

$$16 - 2 = 14$$

What is left?

Now you know these techniques, there are a few more facts left:




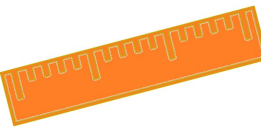
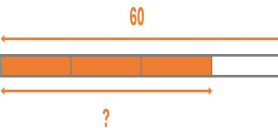




$$5 + 3 = 3 + 5 = 8$$

$$6 + 3 = 3 + 6 = 9$$

$$4 + 7 = 7 + 4 = 11$$

$$5 + 7 = 7 + 5 = 12$$





6.4 Top tips: for Maths exams



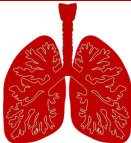



1	Reread TWICE		Re-read the question twice, to understand it better, to spot tricks etc.
2	If you know, DO IT		If you know what to do, go ahead, don't lose time.
3	Not sure, PLAN the steps		If you are not sure, try and spot the different steps for getting to the solution.
4	Be NEAT		Being neat is essential, to avoid getting confused and making mistakes. Use a ruler, compass etc.
5	DRAW & visualise		It helps to draw and visualise the question. It is often enough to understand what to do.
6	ORGANISE the info		It helps to organise the information neatly, either in a list or a table. Be structured, be systematic.
7	Do & write calculations STEP BY STEP	$\begin{array}{r} 5x + z = 17 \\ -z = -2 \\ \hline 5x = 15 \\ 5 \quad 5 \\ \hline 1x = 3 \end{array}$	Make sure you complete your calculation 1 step at a time, clearly marking your results at each step.
8	Check the ANSWER		Check your answer works by replacing it back into the question.
9	Too long or tricky, SKIP	NEXT 	Do not lose too much time on a question. Skip and come back if you have time at the end.
10	At the end, GUESS		At the end, guess the answers to the questions you have not completed. It's worth a try!

6.5 Top tips: managing stress while preparing for, and during an exam

PREPARATION IS KEY TO SUCCESS		
Make TIME		Write yourself a personal study plan: daily goals, revision timetable
Make SPACE		Organise yourself a dedicated study space with your study materials
Get into the MINDSET		Switch off internet, reduce technology and distraction, don't procrastinate or put off the studying

AND KEEP A HEALTHY MIND...		
Do ACTIVITIES you like		Continue to do some of the activities you like, make sure to take time for yourself
Meet FRIENDS, have FUN		Meet and chat with friends and family, have fun.
Take some BREAKS		Take some breaks (max 45 minutes study at a time) and move around
REWARD yourself		Regularly reward yourself after having completed a good amount of work: get a treat, spend time doing something you fancy
Manage STRESS		Explore ways to manage stress: mindfulness, yoga, walk in nature, hot bath. e.g. Insight Timer app, Yoga with Adriene on YouTube
Keep some PERSPECTIVE		Don't put yourself under too much pressure. You are trying your best. It is not forever. Plan fun things for after the exams.
REACH OUT		Talk to someone you trust (family, friend, teacher) and reach out if you are anxious.

...IN A HEALTHY BODY		
SLEEP well		Value your sleep and aim to sleep a minimum of 8 hours per night
EAT well		Eat healthy food and do not skip meals
EXERCISE regularly		Do some sport and exercise regularly
AVOID!		Avoid caffeine, tobacco, and alcohol

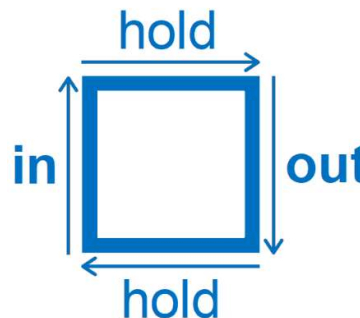
ON EXAM DAY		
SLEEP WELL		It is important to have a good night sleep before an exam. Do not cram the night before. Have a bath and go to bed early.
EAT WELL		Eat lean protein with carbohydrates – it gives energy slowly and keeps you going. Pasta with chicken for breakfast is perfect!
BREATHE & GROUND		Breathing is so important! We often underestimate the importance of breathing exercises to combat stress. While waiting for the exam to start, do some breathing exercises to help bring plenty of oxygen to your brain and calmness to your mind. If you are feeling anxious and nervous, you can also try some grounding exercises
FIND YOUR HAPPY PLACE		If your mind is going 100 mph and you are starting to panic, think of a happy memory and try to feel the joy you felt. Capture this feeling and now, rub your hand, or pinch your arm to bring you back to now, but try to keep the happy feeling with you.
JUST START		It's easier once you start: find an easy question and get started!
PASS BUT COME BACK		If you get stuck, stay calm and move on to the next question, but do come back later

Examples of breathing exercises

Imagine you move around a square, 4 seconds on each side:

- **BREATH IN** 4 seconds
- **HOLD** 4 seconds
- **BREATH OUT** 4 seconds
- **HOLD** 4 seconds

Keep going around the square a few times...



Examples of grounding exercises

The grounding techniques are adapted from [University of Sydney Counselling and Psychological Services](https://www.cpa.org.au/grounding-techniques)

- **Noticing:** look around the room and notice what is around you, notice details.
- **Chair:** pay attention to where and how the chair connects to your body, how it supports you.
- **3 objects:** notice 3 objects in the room and describe them to yourself in a great amount of details.
- **5 senses:** look around the room, and notice 5 things you see, 5 things you hear, etc. Cover all 5 senses: sight, sound, smell, taste and touch.
- **Sounds:** listen carefully and identify and name all the sounds you can hear.
- **Counting:** count backwards from 20 or 50 or 100.
- **Breath to feet:** take a deep breath and as you exhale, imagine breathing out strongly through the soles of your feet. Feel the connection of your feet with the floor. Do this three times.
- **Categories:** choose a category (e.g. colours, shapes, textures), then look around the room and name all the things you can see in that category.